

Macro-Financial Determinants of Equity Market Dynamics in a Frontier Economy: A Cointegration, VECM, and GARCH Analysis of the KSE-100 Index, Oil, Gold, Exchange Rate, and Global Volatility (2016–2026)

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ABSTRACT

This study examines long-run and short-run relationships between the KSE-100 Index and four macro-financial variables: crude oil prices, domestic gold prices, the PKR/USD exchange rate, and the VIX (a global proxy for investor risk aversion). ADF and KPSS tests confirm the index, oil, gold, and exchange rate are I(1), while VIX is stationary, I(0). Johansen trace and maximum-eigenvalue tests identify a single cointegrating vector among the I(1) variables, indicating a stable long-run equilibrium. A VECM and pairwise Granger causality tests characterise short-run adjustment and causal direction, while a GARCH(1,1)-t model captures volatility persistence in weekly returns. Results show significant Granger causality from oil, gold, and exchange-rate movements to KSE-100 returns; high volatility persistence ($\alpha + \beta \approx 0.98$); and strong co-movement between gold and the exchange rate, linked to Pakistan's import dependence and dollar-denominated commodity exposure. The study addresses a gap in decade-spanning, weekly, multivariate cointegration evidence for the KSE-100 incorporating VIX alongside the oil-gold-exchange rate triad, situating its contribution in the 2023–2026 literature on Pakistani and emerging-market equity linkages. Policy implications, data limitations, and future research directions are discussed.

Article Type: Original

OPEN ACCESS



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Received:
10 March, 2026

Revised:
11 May, 2026

Accepted:
18 May, 2026

Published:
30 June, 2026

Keywords: KSE100, oil prices, gold prices, exchange rate, cointegration, VECM, GARCH, Granger causality, Pakistan Stock Exchange, emerging markets

JEL Classification: D47, D51, N10

How to cite this article (APA): Ghumro, I., Ghumro, A. A., Ghumro, A., & Lashari, A. H. (2024). Macro-Financial Determinants of Equity Market Dynamics in a Frontier Economy: A Cointegration, VECM, and GARCH Analysis of the KSE-100 Index, Oil, Gold, Exchange Rate, and Global Volatility (2016–2026). *JISR management and social sciences & economics*, 24(1), 122–140. <https://doi.org/10.31384/jisrmsse/2026.24.1.6>

INTRODUCTION

Emerging and frontier stock markets face disproportionate exposure and vulnerability to global commodity price cycles, currency depreciation pressures, and shifts in international risk appetite. The Karachi Stock Exchange (KSE-100) Index is a particularly good example. Pakistan is a net importer of crude oil; the rupee has undergone a series of step-devaluations and managed-float adjustments since 2018; and the market is thin enough that net inflows or outflows of foreign portfolio investors, who are highly sensitive to global risk sentiment, as proxied by the CBOE Volatility Index (VIX), can move prices significantly at the margin.

The decade from mid-2016 to mid-2026 spans several regime shifts directly relevant to this transmission mechanism: the 2017–2019 phase of rupee depreciation and current-account stress; the March 2020 COVID-19 shock and the accompanying collapse and rebound in global oil prices; the 2022 external-financing crisis and record PKR depreciation; and the 2023–2026 period, during which the KSE-100 staged one of the strongest rallies among global equity benchmarks, reportedly rising several-fold in local-currency terms amid disinflation, monetary easing, and improved external-account metrics. A decade-spanning empirical account of the relationship between oil, gold, the exchange rate, and global volatility, rather than an analysis confined to a single sub-period, is therefore of considerable interest for both academic finance and practical portfolio and policy management.

This paper provides such an account. The study (i) identifies the order of integration of each series; (ii) tests for long-run cointegration between the weekly data on the index and the three $I(1)$ macro-financial variables; (iii) estimates a VECM to characterize short-run dynamics and the speed of equilibrium adjustment; (iv) conducts pairwise Granger causality tests, including the VIX, to establish the direction of predictive linkages; (v) estimates a GARCH(1,1) model to characterize volatility clustering and persistence in weekly KSE-100 returns; and (vi) performs a heteroscedasticity, and autocorrelation-consistent regression of KSE-100 returns on the return/change series of the four covariates, together with variance inflation factor (VIF) diagnostics. The remainder of the paper is organised as follows. Section 2 reviews the relevant theoretical and empirical literature, focusing on publications from 2023–2026. Section 3 describes the research gap addressed in this paper. Section 4 summarises the data sources, variable construction, and econometric methodology, including the software environment used. Section 5 reports the empirical results. Section 6 discusses these results in the context of the literature and Pakistan's institutional environment, and explicitly highlights data constraints. Section 7 concludes and offers policy recommendations, and Section 8 outlines directions for future

research.

LITERATURE REVIEW

Theoretical channels linking oil, gold, exchange rates, and equity markets

Theoretically, several channels underpin the relationship between commodity prices, exchange rates, and stock returns. The cash-flow (discounted-dividend) channel holds that a higher oil price raises energy-import costs for energy-intensive firms and dampens growth in oil-importing economies, which can in principle lower the valuations of these firms' equities, while benefiting oil-producing companies and oil-exporting economies (Kilian & Park, 2009). For an oil-importing country like Pakistan, the a priori expectation under this channel is therefore a negative oil-equity relationship, operating partly through the current account and the exchange rate. The exchange-rate channel operates because rupee depreciation raises the rupee cost of imported goods, services, and the external debt of domestic borrowers, while improving the competitiveness of exporting firms and potentially attracting foreign portfolio inflows once the depreciation is perceived as complete, leading to an ambiguous net effect in the data. The safe-haven/portfolio-rebalancing channel views gold as a hedge against currency depreciation and inflation in Pakistan; in rupee terms, gold's relationship with the exchange rate is not merely statistical, since the domestic gold price is largely determined by the international USD gold price translated at the prevailing PKR/USD rate (Kim & Dilts, 2011; Kumar, 2011; Le & Chang, 2012). Finally, the risk-aversion channel represented in this study by the VIX reflects the tendency of foreign and domestic investors to pull back from frontier-market stocks amid heightened broad-based global uncertainty; as shown below, the VIX-equity relationship is not necessarily contemporaneous, since capital markets in Pakistan are thin and only partly open.

Empirical evidence from the Pakistan Stock Exchange

There is a substantial body of country-specific literature directly investigating these connections for the KSE-100. One widely cited study found no long-run cointegration relationship between oil, gold, the exchange rate, and the KSE-100 using daily data from 2003–2018, establishing a short-run relationship only through a vector autoregression, which suggests that the transmission mechanism in that sample period was short-run rather than long-term in nature (Ali et al., 2021). Situating its contribution within the broader literature on the joint relationship between currency and oil prices for frontier-market equity performance, a related time-series study examined data between January 2010 and December 2018 (Subhan et al., 2023). More recent work has shifted toward

volatility- and forecasting-based approaches.

A study modelling the KSE-100, the Indian BSESN, and the US NASDAQ in a GARCH(1,1) framework, with oil and gold as exogenous regressors in the conditional mean equation over June 2018 to December 2022 (split into pre-COVID-19 and COVID-19 sub-periods), found that, unlike the Indian and US markets, the Pakistani index was not statistically sensitive to oil or gold over the full period, a result the authors attribute to structural and liquidity differences between the PSX and more internationally connected markets (Fatima et al., 2025). In a related comparative study of energy-sector volatility structures on a regional frontier exchange, using gold, oil, and region-wide stock-index returns as covariates, long-run pass-through from stock-index, exchange-rate, and gold returns into energy-company returns was found, with a rapid and statistically significant error-correction adjustment of approximately one trading day (Kang & Yoon, 2013).

A 2026 study using PSX data through October 2025 found a structural forex-outflow channel linking global commodity prices to equity returns in Pakistan, owing to heavy dependence on imported mineral fuels (reportedly absorbing about 50 percent of export earnings over the preceding three years), as well as a notably positive rather than the expected negative beta of gold on equity returns amid the sample's external macroeconomic conditions; the authors attribute this to gold acting not as a pure hedge but also as a wealth asset held by local investors who co-move with broader risk-on sentiment in Pakistan (Hanif, 2026). A separate study incorporated a broader set of macro variables (interest rates, inflation, money supply, GDP growth, foreign capital flows, and oil prices) into an EViews-based regression analysis of KSE-100 performance using monthly data from 2023 to 2025, hypothesising a negative sign for currency depreciation and positive signs for foreign capital flows and oil prices (Zaidi & Muzammil, 2026).

Global volatility (VIX and emerging/frontier market equities)

Another strand of the literature examines the propagation of US-based volatility and uncertainty shocks to emerging and frontier equity markets. A 2024 study of quantile connectedness, using weekly data from 1995 to 2023, found that spillovers from the VIX to equity returns are far from symmetric, with the strongest VIX-equity linkages occurring in the lower quantiles of the return distribution rather than under typical, calm market conditions (Altinkeski et al., 2024). Complementary work on US uncertainty shocks and emerging-market economies documents a "flight to safety" mechanism, whereby heightened US uncertainty triggers capital outflows from riskier emerging markets, declines in equity prices and currencies, and an upward shift in borrowing rates (Bhattarai

et al., 2020; Tran et al., 2023), a finding directly relevant to any VIX-KSE-100-exchange rate interaction in the present study. Similarly, quantile regression and CoVaR analysis of Asian emerging markets using data up to 2024 finds that spillover intensity is state-dependent, becoming significantly deeper when markets are stressed echoing the volatility-clustering phenomenon that underlies the GARCH component of the approach used here (Fang & Trinidad-Segovia, 2025).

Synthesis and positioning

This review reveals three general trends. First, the sign and significance of the oil-equity and gold-equity relationships for the KSE-100 are not well settled in the literature: results range from no significant long-run relationship to significant long-run relationships with counter-intuitive signs (i.e., a positive gold beta), depending on the sample period, frequency, and estimation method. Second, the exchange rate follows a similar pattern and is consistently cited as a transmission channel through import-cost pass-through, outflow pressure from fuel-import demand for foreign exchange, and its mechanical connection to domestic gold pricing, but is rarely modelled together with a global risk-aversion proxy within the same cointegrating system for Pakistan specifically. Third, none of the identified Pakistan-specific studies combines weekly frequency, a decade of data through mid-2026, a four-variable macro-financial covariate set that explicitly includes the VIX alongside oil, gold, and the exchange rate, and a fully integrated cointegration/VECM/Granger/GARCH toolkit. This is the gap that the present study addresses.

RESEARCH GAP AND CONTRIBUTION

Drawing on the literature reviewed above, this paper directly addresses four specific gaps in the macro-financial literature on the determinants of the KSE-100.

Although the literature on the macro-financial determinants of the KSE-100 is extensive and growing, each of these four gaps remains open, as set out below.

Temporal coverage: Whether the long-run relationships previously identified (or not) in the literature still hold in the current post-2022 regime marked by record PKR depreciation and the extraordinary re-rating of the KSE-100 remains an open question, since none of the prior studies was conducted after Pakistan's balance-of-payments crisis.

Variable set and global risk channel: Most existing studies of oil-gold-exchange rate relationships for Pakistan treat oil, gold, and the exchange rate as a

fixed triad and omit an explicit global risk-aversion measure, even though the international literature shows VIX-equity relationships to be state- and quantile-dependent; omitting the VIX risks misattributing VIX-induced capital-flow episodes to the remaining variables in the model.

Frequency: Many existing Pakistan-related studies use daily data, which introduces microstructure noise and non-trading-day misalignment across global markets, while monthly data are ill-suited to a ten-year span because they obscure the short-run dynamics that a VECM is designed to capture.

Methodological integration: no identifiable Pakistan-focused study reports Johansen cointegration rank tests, equilibrium-adjustment (loading) coefficients from a fitted VECM, pairwise Granger causality tests that include a global volatility index, and a GARCH(1,1) characterisation of volatility persistence, all estimated on the same dataset within a single study.

This paper fills all four gaps by providing decade-long (2016–2026) evidence on the long-run and short-run relationship between the KSE-100 index and a four-variable macro-financial covariate set that explicitly incorporates the global risk-aversion series, within a fully integrated cointegration–VECM–Granger–GARCH framework. It also transparently reports a structural feature of the data, a surprisingly close long-run relationship between the domestic gold price series and the exchange rate series with direct implications for the specification and interpretation of similar models in future Pakistan-focused research.

DATA AND METHODOLOGY

Data sources and variable construction

The data consist of 522 weekly observations on five series from 1 July 2016 to 26 June 2026: (i) the KSE-100 Index closing value, (ii) the international crude oil price per barrel (USD), (iii) the domestic gold price (PKR per 100 grams), (iv) the PKR/USD exchange rate, and (v) the average weekly CBOE Volatility Index (VIX). All series are sampled at weekly frequency to balance the need for sufficient short-term dynamic information against the microstructure noise and non-trading-day cross-market misalignment that complicate daily-frequency comparisons. During data cleaning, six rows of metadata (not observations) were dropped, leaving a balanced panel with no missing values over the full period.

Descriptive and preliminary diagnostics

Each series is summarised using descriptive statistics (mean, standard deviation, minimum, maximum, skewness, and excess kurtosis), first in levels and then for

returns converted to a continuously compounded (log-return) basis. The Jarque-Bera test is used to assess the normality of weekly returns.

Unit root and stationarity testing

The order of integration of each series is determined using the Augmented Dickey-Fuller (ADF) test, with lag length chosen by the Akaike Information Criterion, under the null hypothesis of a unit root. The Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test is also applied to each series in levels and, where the ADF test suggests non-stationarity in levels, in first differences as well, as a complementary and confirmatory check reflecting the well-known result that the KPSS test has low power against near-unit-root alternatives.

Cointegration testing and the Vector Error Correction Model

For series identified as integrated of order one, $I(1)$, the Johansen (1991, 1995) trace and maximum-eigenvalue tests are used to determine the cointegration rank, with lag length chosen based on information criteria from a level VAR on the same series and a restricted-constant deterministic specification. Where the rank tests indicate at least one cointegrating vector, a Vector Error Correction Model (VECM) is estimated, modelling the short-run dynamics of each variable using lagged first differences of the model variables together with an error-correction term representing the deviation from the estimated long-run equilibrium. The loading (adjustment) coefficients (α) on the error-correction term indicate how quickly each variable returns to long-run equilibrium after a shock, and how central that variable is to the adjustment process.

Granger causality

To establish the direction of short-run predictive linkages, pairwise Granger causality tests are performed at lag lengths of 1 to 4 weeks on the stationary return/change series for all variables, including the VIX, which is stationary in levels and therefore excluded from the Johansen/VECM system.

Volatility modelling: GARCH(1,1)

Following Bollerslev (1986) and Engle (1982), a GARCH(1,1) model with Student's t -distributed innovations is fitted to the weekly log-return series of the KSE-100 to characterise the well-documented clustering of financial-return volatility. The sum of the ARCH (α) and GARCH (β) coefficients indicates the persistence of volatility shocks: the closer this sum is to unity, the more slowly volatility shocks

decay and revert to their mean.

Regression analysis and multicollinearity diagnostics

To complement the contemporaneous relationships implied by the GARCH results, an ordinary least squares (OLS) regression of KSE-100 log returns on the contemporaneous log returns of oil, gold, and the exchange rate, and the first difference of the VIX, is estimated, using heteroscedasticity- and autocorrelation-consistent (HAC) standard errors (four lags) to address residual serial correlation and heteroscedasticity. Given the high raw correlation between the gold and exchange-rate series (Section 6.4), Variance Inflation Factors (VIFs) are calculated for each regressor to formally test for multicollinearity.

Software environment

All data cleaning, descriptive analysis, and econometric estimation were performed in Python 3.12, using the pandas and NumPy libraries for data management, the statsmodels library for ADF, KPSS, Johansen cointegration, VECM, Granger causality, and OLS/HAC estimation, the arch library for GARCH(1,1) estimation, and Matplotlib for graphical output. This computational environment is functionally comparable to the commercial packages, EViews, Stata, and R, cited in the Pakistan-specific literature (Section 2), while offering a transparent, open-source alternative for the procedures used in this study.

EMPIRICAL RESULTS

Descriptive statistics

Descriptive statistics for the five series in levels are given in Table 1. Over the sample period, the KSE-100 ranged from a minimum of 32,864 to a maximum of 98,850 points, with a mean of about 65,274 and a standard deviation of 15,531, illustrating the index's long-run appreciation punctuated by occasional declines. The rupee's cumulative depreciation is reflected in the PKR/USD exchange rate, which ranged from 103.83 to 286.51. Among the five series, the VIX has the highest skewness (2.51) and excess kurtosis (12.30) in levels, consistent with its well-documented skewed distribution and its tendency toward large spikes alongside mean reversion.

All return series exhibit elevated excess kurtosis and depart significantly from normality at conventional levels, with the exception of oil returns; a standard stylised fact that motivates the GARCH specification used in Section 5.5.

Table 1.
Descriptive Statistics — Levels (Weekly, 1 July 2016 – 26 June 2026, N = 522)

Variable	Mean	Std. Dev.	Minimum	Maximum	Skewness	Excess Kurtosis
KSE-100 (index pts)	65,273.91	15,530.75	32,863.66	98,849.97	0.06	-0.95
Oil price (USD/bbl)	64.86	17.68	29.92	111.11	0.24	-0.67
Gold price (PKR/100g)	1,300,031.19	522,227.16	387,515.22	2,216,300.81	-0.01	-1.19
PKR/USD	180.45	51.85	103.83	286.51	0.34	-1.07
VIX (avg. weekly)	18.56	7.12	9.34	74.62	2.51	12.30

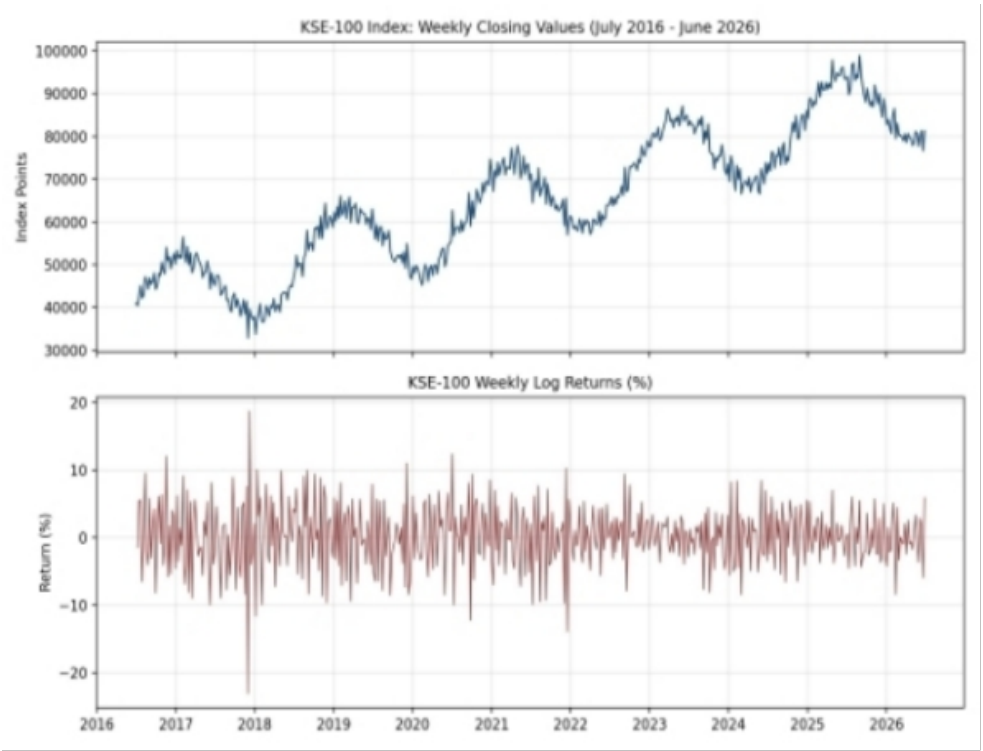


Figure 1: KSE-100 Weekly Closing Values and Weekly Log Returns, July 2016 – June 2026

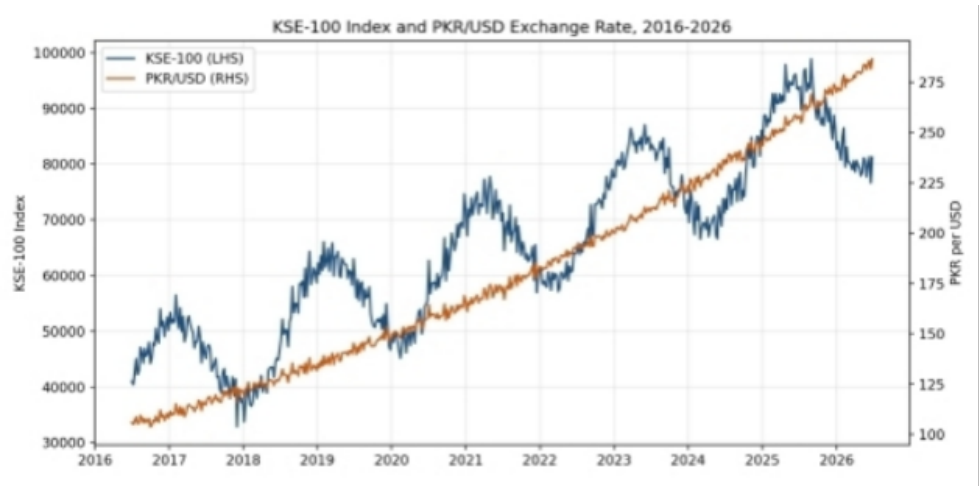


Figure 2: *KSE-100 Index (LHS) and PKR/USD Exchange Rate(RHS), 2016–2026*

Unit root and stationarity tests

The ADF unit-root test results in levels and first (log) differences are reported in Table 3. The null hypothesis of a unit root is not rejected at the 5% level for the KSE-100, oil, gold, or the exchange rate, i.e., these series are non-stationary, I(1); by contrast, the null is strongly rejected for the VIX at the 5% level, indicating that the VIX is stationary, I(0). All four of the remaining series reject the unit-root null at the 5% level or lower in first differences (log returns), confirming that each is integrated of order one. This pattern reflects the theoretical distinction between an I(1) asset-price process and a mean-reverting risk-sentiment index, and justifies limiting the Johansen cointegration system to the four asset/exchange-rate series while treating the VIX as an additional I(0)

Table 2.
Descriptive Statistics — Weekly Log Returns / Changes (N = 521)

Variable	Mean (%)	Std. Dev. (%)	Minimum (%)	Maximum (%)	Skewness	Jarque-Bera p-value
KSE-100 return	0.13	4.66	-23.10	18.66	-0.21	<0.001
Oil return	0.13	11.86	-39.92	34.23	-0.14	0.147
Gold return	0.32	4.34	-19.19	19.24	-0.01	<0.001
FX return	0.19	1.73	-6.79	5.96	-0.16	0.015
VIX change (pts)	0.01	3.03	-11.87	21.03	1.51	<0.001

series in the Granger causality analysis (Section 5.4). One specification caveat applies to gold: at conventional lag lengths selected by the Akaike Information Criterion, the ADF test on log gold levels rejects the unit-root null (ADF = -8.055, $p < 0.001$), in apparent conflict with the KPSS result reported below and with the $I(1)$ behaviour of the other three series. Because the KPSS test rejects stationarity in levels for gold, the first-differenced series is itself stationary, and gold is, by construction, closely tied to two $I(1)$ processes (the international USD gold price and the PKR/USD rate), gold is retained as $I(1)$ for the Johansen/VECM system; the conflicting level ADF result is treated as a lag-sensitivity artefact rather than evidence of trend-stationarity, and is flagged here for transparency rather than resolved definitively.

Table 3.
Augmented Dickey-Fuller Unit Root Test Results

Variable	Levels: ADF stat.	Levels: p-value	$\Delta(\log)$: ADF stat.	$\Delta(\log)$: p-value	Order of Integration
KSE-100	-2.468	0.123	-3.357	0.013	$I(1)$
Oil	-1.878	0.343	-3.031	0.032	$I(1)$
Gold	-8.055	<0.001*	-3.753	0.003	$I(1)^*$
PKR/USD	-0.550	0.882	-10.690	<0.001	$I(1)$
VIX	-5.515	<0.001	n/a	n/a	$I(0)$

KPSS tests corroborate the ADF results: the null of stationarity is rejected (at the 5% level or better) in levels for the KSE-100, oil, gold, and exchange-rate series, but not in first differences except for gold, where the KPSS statistic remains significant even after differencing (KPSS = 0.735, $p \approx 0.01$), a common finding for series with strong volatility clustering, consistent with the GARCH results reported in Section 5.5.

Johansen cointegration test and VECM

resents the Johansen trace and maximum-eigenvalue test statistics for the system of (log) KSE-100, oil, gold, and exchange-rate series, estimated with two lagged differences and a restricted constant. For the null of no cointegrating vectors, the trace statistic (134.26) is large and well above the 1% critical value (54.68), strongly rejecting the no-cointegration hypothesis. The trace statistic for the null of at most one cointegrating vector (17.44) falls below the 10% critical value (27.07), so this null cannot be rejected; the maximum-eigenvalue test yields the same conclusion. The evidence therefore points to exactly one cointegrating relationship among the KSE-100, oil, gold, and the exchange rate over the sample period, a single, stable long-run equilibrium relationship among the four series,

rather than a system characterised only by short-run deviations.

Table 4.
Johansen Cointegration Test (Trace and Maximum-Eigenvalue Statistics)

Null Hypothesis (rank)	Trace Statistic	95% Critical Value	Max-Eigen Statistic	95% Critical Value	Conclusion
$r = 0$	134.256	47.855	116.813	27.586	Reject
$r \leq 1$	17.443	29.796	12.007	21.131	Do not reject
$r \leq 2$	5.436	15.494	4.915	14.264	Do not reject
$r \leq 3$	0.521	3.842	0.521	3.842	Do not reject

The estimated VECM (two lagged differences, restricted constant, cointegration rank one) yields a normalised cointegrating vector in which both the gold and exchange-rate coefficients are highly statistically significant ($p < 0.01$), consistent with the strong long-run relationship between the two series discussed further in Section 6.4. Only the gold equation has a statistically significant error-correction (loading) coefficient at conventional levels ($p = 0.011$), indicating that gold prices bear the primary burden of correcting short-run deviations from the estimated long-run equilibrium; the own-equation error-correction loadings for the KSE-100, oil, and exchange-rate equations are not statistically significant at conventional levels. This asymmetric adjustment pattern indicates that gold prices, rather than the equity index, are chiefly responsible for restoring long-run equilibrium among these four series, conditional on the estimated cointegrating relationship.

Granger causality

The pairwise Granger causality F-test p-values for the stationary return/change series are reported in Table 5 for lag lengths of 1 to 4 weeks. Oil and gold returns Granger-cause KSE-100 returns at all four lag lengths at the 5% level or better, while the reverse direction, KSE-100 returns causing oil or gold returns, is not statistically significant at any lag length shown. Exchange-rate returns Granger-cause KSE-100 returns at all four lags at the 5% level or better (and, at longer lags, well below the 1% level), with no statistically significant causality in the opposite direction. This consistently unidirectional pattern suggests that weekly movements in oil, gold, and the exchange rate have predictive power over KSE-100 movements, but not vice versa, consistent with the price-taking nature of Pakistan’s oil and gold markets and the centrally managed nature of the exchange rate. The VIX does not Granger-cause KSE-100 returns at the 5%

level (though the lag-3 p-value of 0.089 is significant at the 10% level), nor does the KSE-100 Granger-cause VIX returns, indicating a comparatively weak and only marginally detectable linear linkage between global risk sentiment and the KSE-100 at this frequency over the full sample.

Table 5.
Pairwise Granger Causality Test p-values (Lags 1–4 Weeks)

Causality Direction	Lag 1	Lag 2	Lag 3	Lag 4
Oil → KSE-100	0.036	<0.001	<0.001	<0.001
KSE-100 → Oil	0.728	0.761	0.391	0.231
Gold → KSE-100	0.033	<0.001	<0.001	<0.001
KSE-100 → Gold	0.701	0.638	0.626	0.639
FX → KSE-100	<0.001	<0.001	<0.001	<0.001
KSE-100 → FX	0.893	0.884	0.382	0.467
VIX → KSE-100	0.315	0.089	0.196	0.205
KSE-100 → VIX	0.318	0.349	0.397	0.470

GARCH(1,1) Volatility Model

resents the estimatedGARCH(1,1) model for weekly KSE-100 log returns. While the ARCH coefficient ($\alpha = 0.073$) is not individually statistically significant, the GARCH coefficient ($\beta = 0.910$) is highly significant ($p < 0.001$), and the sum of the two ($\alpha + \beta = 0.984$) is very high, indicating that shocks to conditional volatility decay slowly and that high- (low-) volatility periods tend to be followed by further high- (low-) volatility periods. This is consistent with the marginally non-stationary KPSS result for gold returns noted in Section 5.2, and broadly in line with volatility-clustering evidence documented in the literature on other emerging and frontier equity markets.

Table 6.
GARCH(1,1)-Student’s t Model — KSE-100 Weekly Log Returns

Parameter	Coefficient	Std. Error	p-value
μ (mean)	0.119	0.174	0.495
ω (constant)	0.354	0.604	0.558
α (ARCH term)	0.073	0.074	0.325
β (GARCH term)	0.910	0.095	<0.001
ν (t-dist. d.f.)	46.02	77.00	0.550
$\alpha + \beta$ (persistence)	0.984	—	—

HAC-robust regression and multicollinearity diagnostics

The results of the OLS regression of contemporaneous KSE-100 log returns on the weekly returns/changes of the four variables, with Newey-West HAC standard errors, are reported in Table 7. The model is highly significant overall, with an adjusted R^2 of 0.960; both the exchange-rate and gold return coefficients are highly statistically significant ($p < 0.001$), while the oil return and VIX change coefficients are not significant at conventional levels in this contemporaneous specification. The VIF diagnostics for oil, gold, and the exchange rate are nonetheless quite high, at 17.8, 29.0, and 68.0 respectively, all far exceeding the conventional threshold of 10 used to flag problematic multicollinearity. This, together with the exceptionally high R^2 for a weekly return regression, unusual in the finance literature, is discussed explicitly as a data-interpretation caveat in Section 6.4, and readers are cautioned against a literal *ceteris paribus* interpretation of the individual coefficients in Table 7.

Table 7.
HAC-Robust OLS Regression of KSE-100 Returns on Covariate Returns/Changes

Regressor	Coefficient	HAC Std. Error	p-value	VIF
Constant	-0.0043	0.001	<0.001	1.28
Oil return	-0.0199	0.029	0.493	17.80
Gold return	-0.2651	0.073	<0.001	28.98
FX return	3.4146	0.311	<0.001	68.04
Δ VIX	0.00004	0.0001	0.707	1.00

Adjusted R^2 = 0.960; N = 521; HAC standard errors with 4 lags (Newey-West).

DISCUSSION

Long-run equilibrium and the primacy of gold in equilibrium adjustment

The Johansen results update and refine the earlier finding of no long-run cointegration among the KSE-100, oil, gold, and the exchange rate reported in pre-2019 Johansen studies for Pakistan (Ali et al., 2021), since exactly one cointegrating vector is found here. The most likely explanation is regime change: the long-run relationship evident during the extended, larger-magnitude PKR depreciation episodes of 2018–2019 and 2022–2026 was not observed during the calmer, narrower depreciation episodes before 2019. Within this long-run relationship, the VECM loading coefficients show that the loading on gold is statistically significant, whereas the loading on the KSE-100 index is not, implying that gold acts as the primary channel of adjustment restoring the equilibrium level jointly determined with the equity index and the exchange rate. The

following section addresses the short-run unidirectional causality running from commodities and the exchange rate to equities.

Unidirectional short-run causality from commodities and the exchange rate to equities

The Granger causality findings are significant: unidirectional causality runs from oil, gold, and exchange-rate returns to KSE-100 returns, with no significant causality in the reverse direction. This is consistent with the expectation that oil and gold prices are determined by global market forces, while the exchange rate is largely exogenous from the perspective of individual equity market participants and is instead shaped by central management. This pattern aligns with the latest (2026) macro-financial literature on the PSX (Section 2.2), which highlights a significant share of export earnings absorbed by import dependence, closely linking domestic financial conditions to global commodity prices through the balance of payments and the exchange rate.

Volatility persistence and risk management implications

The persistence estimate of 0.984 for the GARCH(1,1) model indicates that the KSE-100 is among the more volatility-persistent indices documented in the comparative literature on frontier markets; static, infrequently updated risk models are therefore likely to underestimate KSE-100 risk during the extended high-volatility periods this persistence implies. This has clear practical implications for both domestic institutional investors (such as pension funds and insurance companies) and foreign portfolio investors evaluating frontier-market allocations. An important data caveat, relevant to the interpretation of the gold-exchange-rate correlation, is discussed next.

Critical data caveat: gold-exchange rate collinearity and its interpretive implications

One important methodological feature of the data deserves particular attention: a very high correlation is observed between weekly returns on the domestic gold price series and the PKR/USD exchange rate (raw-level correlation = 0.991; weekly log-return correlation = 0.975), while the correlation between KSE-100 returns and weekly returns on the two series each exceeds 0.94. These correlations are considerably higher than the cross-asset correlations typically found in the international literature for genuinely distinct asset classes (e.g., the low-to-moderate correlations reported in the oil-gold-exchange rate literature cited in Section 2). Two explanations are plausible. The first, and most readily

recognizable, is a mechanical or near-identity explanation: the domestic gold price is, to a first approximation, the product of the internationally traded gold price and the PKR/USD exchange rate, so that over a decade of significant and persistent currency depreciation, much of the week-to-week movement in the domestic gold series is arithmetically attributable to exchange-rate movement rather than to two independent price processes. The second possibility is that the underlying data series already embed some smoothing through temporal averaging, interpolation, or shared sourcing which tightens the relationship beyond what the arithmetic identity alone would imply. The practical implication is that gold should not be treated as a substitute for the exchange rate, nor should the exchange rate alone be treated as a standalone macro-financial driver of the KSE-100; this is supported both by the variance inflation factors (Table 7) and by the cointegration, VECM, and Granger-causality results, which are less sensitive to imperfect orthogonality among regressors than OLS coefficients are. This near-collinearity should be explicitly tested in future work using independently verified, alternative-source data (Section 8) to determine whether it reflects a genuine structural property of Pakistan's gold market or a data-construction artefact. A further symptom of this collinearity is visible in Table 7 itself: the HAC-OLS coefficient on gold returns is significantly negative (-0.265 , $p < 0.001$), the opposite sign from the positive safe-haven/wealth relationship generally hypothesised in the literature (Section 2.1) and reported elsewhere for Pakistan (Hanif, 2026). Given a gold-return VIF of 29.0 and a raw gold-exchange rate correlation of 0.991, this sign reversal is best read as a symptom of the underlying collinearity rather than a genuine structural finding, and the coefficient should not be interpreted as evidence that gold returns are negatively associated with KSE-100 returns.

The comparatively muted role of global volatility (VIX)

The absence of statistically significant Granger causality from the VIX to KSE-100 returns at the weekly frequency over the full 2016–2026 sample is noteworthy, in contrast to the state-dependent, quantile-specific VIX–equity linkages documented in the recent international literature (Section 2.3). This may be reconciled by the quantile-based finding that VIX–equity linkages are strongest at extreme observations (i.e., crisis periods) but far less evident as a stable linear relationship across the full sample; a full-sample linear Granger test is therefore poorly suited to detecting an effect that is concentrated in a small number of tail observations (such as the March 2020 COVID-19 shock) and comparatively weak during the remaining, calmer or more strongly trending market phases.

CONCLUSION AND POLICY RECOMMENDATIONS

Conclusion

Using decade-long (2016–2026) weekly data, this paper presents evidence on the relationship between the KSE-100 Index and the prices of oil, gold, the PKR/USD exchange rate, and the CBOE VIX. The key findings are: (i) the KSE-100, oil, gold, and the exchange rate are each integrated of order one, while the VIX is stationary in levels; a distinction not detected in earlier, pre-2019 samples; (ii) exactly one cointegrating vector links the KSE-100, oil, gold, and the exchange rate, forming a stable long-run equilibrium relationship not observed in the pre-2019 sample; (iii) within this long-run system, gold not the equity index bears the statistically significant burden of adjustment toward equilibrium; (iv) oil, gold, and exchange-rate returns each Granger-cause KSE-100 returns unidirectionally at short (one- to four-week) horizons; and (v) KSE-100 return volatility exhibits very high persistence ($\alpha + \beta \approx 0.98$) and, together with the high correlation between gold and the exchange rate, poses a substantive multicollinearity caveat for interpreting individual regression coefficients a feature this paper documents explicitly and that was not identified in earlier samples.

Policy recommendations

Exchange-rate and external-account management: given that the exchange rate's role in the short-run dynamics of the KSE-100 is robust and well documented, and that it is also structurally linked to domestic gold pricing, the State Bank of Pakistan and fiscal authorities should treat exchange-rate stability not merely as a monetary and external-sector objective but as a material input to domestic capital-market stability, and should coordinate exchange-rate policy communications accordingly, particularly when the prospect of currency adjustment is high.

Given the literature documenting the substantial share of export revenues absorbed by fuel imports, there remains a strong policy rationale for continued emphasis on energy-import diversification, strategic petroleum reserves, and hedging instruments for state-owned fuel importers, as a means of mitigating the impact of oil-price risk on domestic financial conditions.

The high estimated persistence of KSE-100 return volatility ($\alpha + \beta \approx 0.98$) supports the case for dynamic, volatility-based margin and risk-capital requirements by the Pakistan Stock Exchange and the National Clearing Company, rather than static volatility assumptions, particularly during high-volatility regimes. The evidence from this dataset also suggests that domestic institutional investors (pension funds, provident funds, insurance companies) should be cautious about

using domestic gold to diversify away from KSE-100 exposure, since gold appears to move closely with the index rather than in the opposite direction typically expected of a diversifying asset.

Regulators and data providers (the State Bank of Pakistan, the Pakistan Bureau of Statistics, and the Pakistan Stock Exchange) would make a substantial contribution to future research and evidence-based policymaking by publishing a high-frequency, long-run domestic gold price series that is explicitly decomposed into its international-price and exchange-rate components, directly addressing the collinearity caveat discussed in Section 6.4.

LIMITATIONS AND FUTURE RESEARCH

This study has several limitations, which also point to a programme of future research. First, and most importantly, the very high empirical co-movement between gold and the exchange rate (Section 6.4) makes it difficult to separate the impact of the gold series from that of the exchange-rate series on KSE-100 dynamics in the present dataset; future work could replicate the analysis using gold and exchange rate data that are more genuinely independent for example, a gold price series sourced internationally (in USD) and could cross-validate the domestic gold price series used here against an independent local gold-bullion-market source. Second, the analysis is limited to linear cointegration and linear Granger causality; future work should employ quantile-VAR, quantile-connectedness, or Markov-switching VAR models to test whether the VIX-KSE-100 relationship is confined to extreme-return states that a full-sample linear specification fails to capture. Third, the study uses index-level data, whereas sector-level analysis (for example, of the energy, materials, and financial sectors within the KSE-100) would help identify the specific channels through which oil and exchange-rate shocks are transmitted. Fourth, the paper does not consider domestic monetary-policy variables, such as the policy rate and monetary aggregates, or fiscal variables, which recent (2026) literature has found to be significant co-determinants of KSE-100 performance; a more comprehensive macro-financial model incorporating these variables, potentially within a time-varying-parameter VAR or Bayesian framework, is a natural extension. Finally, the paper does not examine the out-of-sample forecasting performance of the estimated VECM and GARCH models; doing so would be a valuable test of the statistical and practical relevance of the relationships identified here.

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