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Forecasting the stock market returns using nonlinear hybrid GARCH-SETAR model: Empirical study for Pakistani stock markets

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ABSTRACT

Forecasting stock market returns is a valuable tool for investors seeking to enhance their gains in stock trading. Predicting stock prices proves to be a formidable endeavor due to its substantial volatility, non-linear characteristics, trends, and responsiveness to multifaceted variables, including economic conditions, market trends, seasonality, and sentiment. Despite these complexities, non-linear methodologies like threshold time series and conditional heteroscedasticity models are underutilized. This study aims to assess the predictive capabilities of a hybrid GARCH-SETAR model in the context of stock market returns, encompassing both Islamic and conventional stocks listed on the Pakistan Stock Exchange. The Islamic and conventional stock markets in Pakistan, represented by KMI-30, KSE-30 and KSE-100, contain daily data from January, 2012 till June, 2023. Ultimately, it is revealed that the one-step-ahead recursive forecast performance of the hybrid GARCH-SETAR model outperforms other selected linear and non-linear volatility models for the KMI-30, KSE-30 and KSE-100 stock markets based on RMSE, MAE, MAPE and SMAPE forecast evaluation criteria. The empirical findings of this study hold practical significance, as they can be utilized by both local and international investors in making investment decisions, and by policymakers in shaping the financial policies. Specifically, the hybrid GARCH-SETAR model is recommended as an alternative approach to obtain optimal forecasts for Islamic and conventional stock markets of the PSX.

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INTRODUCTION

Predicting future stock market returns serves as a crucial tool for managing risk and diversifying investment portfolios effectively. The literature contains various forecasting techniques aimed at obtaining precise forecasts to support investment decisions. While numerous empirical studies have utilized these methods to explore the returns of individual stock indices, limited attention has been given to analyzing groups of stock markets or indices. Previous research suggests that a universal forecasting method applicable to all stock markets does not exist. Against this backdrop, the primary objective of this study was to assess the predictive capabilities of different models, including linear, non-linear, threshold and hybrid models, in forecasting stock returns. In the literature, numerous empirical studies can be found evaluating these models by examining daily stock market returns from selected different indices in developed, emerging, and frontier markets across various time periods. The results revealed that no single time series or econometric model could be uniformly applied to accurately forecast all stock markets (Mallikarjuna & Rao, 2019). Stock markets exhibit pronounced volatility, intricate dynamics and dynamism. It is observed that the fluctuations in the stock markets are usually shaped by various factors. These factors included global events i.e. COVID-19 pandemic and macroeconomic conditions etc. From the vast literature, a positive correlation has been found between financial stock markets and economic growth based on theoretical and empirical evidence (Guptha and Rao (2018); Opoku et al. (2019). The profitability of investments in stock markets is greatly reliant on the ability to forecast stock movements accurately. When a forecasting modelling technique can effectively predict market directions, it becomes possible to reduce investment risk and uncertainty. Due to random, unexpected and highly volatile features of the stock market, the forecasting of the stock market returns has become an increasingly attractive, demanding and daunting task over the last three decades Fraz et al. (2022a). Recognizing the capabilities of various forecasting models and determining the most effective model among competitors is crucial in the policy-making process. Consequently, a range of models for estimating and predicting economic variables has been developed. A suitable risk model should be capable of accommodating the inherent characteristics of financial returns. Recent academic research has highlighted that many financial assets display structural breaks in their volatility patterns. Neglecting this aspect can significantly impact the accuracy of volatility predictions, as observed (Ardia et al., 2018). This study explores the forecast performance of benchmark models namely linear Box-Jenkins methodology ARIMA model, nonlinear ARIMA-GARCH, self-exciting threshold autoregressive (SETAR) and proposed hybrid GARCH-SETAR model, which combines conditional variance feature of GARCH and nonlinear threshold feature of SETAR models respectively. Recently, the combination of SETAR and GARCH models has been used to model and forecast various macroeconomic financial variables i.e. daily streamflow water river, cryptocurrencies, global stock markets etc (Siu and Elliott (2021) ; Guo et al. (2021)) (Bildirici et al., 2023; Fathian et al., 2019; Hamida & Scalera, 2019). There is a lack of extensive literature focusing on the modelling and forecasting of SETAR and GARCH models specifically based on stock market returns.

The Pakistan Stock Exchange (PSX) is one of the leading stock markets in the world. One of the distinctive features of the PSX is that it hosts both conventional and Islamic stock markets. Research in the field of Islamic finance has consistently aimed to explore whether the returns achieved by the investors in Islamic indices differ from those of conventional index investors. Within the PSX, three prominent indices stand out: the KSE-100, KSE-30, and KMI-30. Comparing multiple stock market indices within a specific economy provides valuable insights into diverse market conditions across different regions of a country. This information is invaluable to entrepreneurs, investors, and the general public, helping them make informed decisions for optimal trading and investment outcomes. In July 2008, the introduction of the Islamic index, known as KMI-30, was a significant milestone. It was created to offer a platform for ethical investors seeking alignment between their financial goals and religious principles and values. On the other hand, the KSE-30 index, operating under the Free Float methodology, comprises the 30 most liquid businesses listed on the Pakistan Stock Exchange. It commenced operations on September 1, 2006. Meanwhile, the Karachi Stock Exchange 100 Index (KSE-100), established on August 14, 1947, remains a prominent component of the PSX. It serves as Pakistan's primary stock exchange index, encompassing the largest 100 companies based on market capitalization and representing all sectors of the PSX. The KSE-100 index also serves as a benchmark for price comparisons on the Pakistan Stock Exchange (PSX), calculated using the Free Float Market Capitalization approach.

It can be observed from a huge literature review that there is no study done yet in which the features of nonlinearity and volatility are studied and forecast using a mixture of nonlinear and volatility models i.e. hybrid SETAR-GARCH model. In the case of stock markets namely conventional KSE-100, KSE-30 and Islamic KMI-30 under the Pakistan Stock Exchange (PSX), none of the studies has been found in which nonlinear SETAR and GARCH models are used separately or combined to evaluate the forecast performance. In this study, the forecast performance of all-time series models is evaluated using stock returns from both Islamic and conventional stock markets. Following the introduction, section 2 presents the literature review, Section 3 outlines the data and methodology, results and discussions are presented in section 4 and section 5 provides the conclusion.

LITERATURE REVIEW

There is an extensive literature present related to the forecast performance of many linear, nonlinear and hybrid time series models. Siu and Elliott (2021) explored the cryptocurrency pricing of Bitcoin options, aiming to integrate conditional heteroscedasticity and regimes within the returns of Bitcoin. They utilized a combination of the nonlinear SEATR and the Heston-Nandi GARCH model to capture the dynamics of Bitcoin returns. Moreover, Naik and Mohan (2021) compared the GARCH model with the non-linear self-exciting autoregressive (SETAR) model to capture the volatility of Nifty-50 share stocks from January, 2007 to April, 2021. They found that no individual time series model could effectively capture the share market's volatility. Consequently, they introduced the Markov-regime switching and GARCH model (MS-GARCH) for share market forecasting, which outperformed the non-linear time series SETAR model and the traditional GARCH model, as demonstrated through RMSE and MAPE metrics. Midilic (2020) used a hybrid STAR-GARCH model which was a combination of smooth transition autoregressive and GARCH models to improve the forecast performance. He used daily US Dollar/Australian Dollar and FTSE stock returns. He showed that the hybrid model using the features of nonlinearity and conditional variance can give better results. Fathian et al. (2019) employed daily streamflow data from the rivers within the Zarrineh Rood Basid, situated south of Lake Urmia, covering the period from January 1, 1997, to December 31, 2011. They applied SETAR models in conjunction with the ARCH approach. They revealed that the hybrid SETAR-GARCH models outperform traditional approaches and are a suitable choice for modeling and forecasting streamflow data. Mustapa and Ismail (2019) also studied and compared the forecast performance of ARIMA and hybrid ARIMA-GARCH models based on MAE and RMSE forecast evaluation criteria. They found that the forecasting ability of hybrid ARIMA-GARCH models is better than the ARIMA models. They worked on S&P-500 monthly stock returns from 2001 to 2017. M Mallikarjuna and Rao (2019) studied twenty-four stock market returns distributed into the frontier, developed and emerging markets from different countries including a few G7 countries. They used daily closed prices from January 2000 till December 2018. Their main objective was to explore which forecast model is best for each stock market return. They compared the forecast performances of linear ARIMA, nonlinear SETAR, artificial neural network ANN, frequency domain SSA (singular spectrum analysis) and hybrid HM models. They revealed that none of the single or hybrid models is best for all the selected stock market returns. Additionally, they found that the forecast performance of the nonlinear SETAR model is best for the ten, ARIMA for seven while hybrid for only five selected stock market returns based on mean error (ME), RMSE, MSE, MAE and MAPE forecast evaluation criteria. Selmi et al. (2014) used daily S&P-500 futures price data from July 8, 1996 to May 26, 2006 to evaluate the forecast performance of linear ARIMA, nonlinear GARCH, EGARCH, GARCH-M, machine learning ANN and hybrid ARIMA-ANN models based on RMSE, MAPE and Theil IC forecast evaluation criteria. They found that the forecast performance of hybrid ARIMA-ANN models outperforms other individual linear and nonlinear models.

Fraz et al. (2022a) also evaluated the forecast performance of standard GARCH, Component GARCH, Markov regime switching and machine learning LSTM models. They used daily data of KSE-100 and SSE-100 stock market returns from December 2014 till December 2021. They revealed that the forecast performance of machine learning LSTM model is better than volatility models namely GARCH, CGARCH and nonlinear MRS model based on RMSE, MAE, MAPE and SMAPE forecast evaluation criteria. Similarly, Batool et al. (2022) compared the forecasts of traditional GARCH and neural network auto regressive model. They used daily data of KSE-100 stock index from January 2000 till July 2019. They concluded that the hybrid model has the least RMSE value as compared to other models. Moreover, Omar et al. (2022) used three different time periods i.e. pre-Covid-19, during Covid-19 and whole time period to find the best forecast model between the machine learning autoregressive deep neural network (AR-DNN), and random forest (AR-RF). They used Box-Jenkins ARIMA model as a benchmark model. They used KSE-100 daily closing prices of PSX from January 2001 till August 2021. They found that the forecast performance of the AR-DNN models outperforms ARIMA models for whole and pre-Covid-19 pandemic while AR-RF model outclass ARIMA models for during Covid-19 pandemic. Their results are based on correlation coeeficient and forecast evaluation criteria's. According to Aamir et al. (2020) compared the artificial intelligence machine learning ANN and SVM models with the linear NAÏVE, ARIMA and nonlinear GARCH models. They used daily opening KSE-100 stock market prices from January, 2016 till July, 2020. They found that the ANN model can capture the nonlinearity of KSE-100 stock returns. Furthermore, they also revealed that the forecasting ability of ANN model outperforms all other models included GARCH model based on Diebold-Mariano test and forecast evaluation criteria's namely directional statistics (DS), MAPE, RMSE, and MAE. Moreover, Fraz et al. (2020) also evaluated the forecast performance of nonlinear SETAR and Markov regime switching models based on RMSE, MAE and MAPE forecast evaluation criteria. They used different macroeconomic factors annual data with different time domains. They revealed that the SETAR model has a better forecast ability as compare to linear AR and ARIMA models. Devi (2018) applied GARCH models to assess forecast performance using data from the New York Stock Exchange (NYSE) in the USA and the Financial Times Stock Exchange (FTSE) in the UK from January, 1991 to December, 2014 (monthly). Her findings indicated that the EGARCH model exhibited superior performance in terms of RMSE compared to other GARCH models. Additionally, her research revealed that GARCH models were more suitable for the NYSE than the FTSE. R Rizwan and Khursheed (2018) studied the volatility of Islamic stock market returns namely KMI-30 from September 1^{st} 2008 to May 2^{nd} 2012. They used various lags to build Box and Jenkins ARIMA and standard GARCH models. According to their findings, the GARCH (1,1) model is found to be the best to forecast the daily KMI-30 stock market returns as compared to other ARIMA and GARCH models with different lags. Their results are based on MAE, MAPE and RMSE forecast evaluation criteria's. Mubarik and Javid (2016) evaluated the forecast performance of various GARCH models, including GARCH, EGARCH, GARCH in mean (GARCH-M), GJR-GARCH, threshold ARCH (TARCH), and PARCH. They employed data from the KSE-100 stock market returns spanning from July, 1998 to June, 2011. The evaluation of GARCH models considered different statistical distributions. Their results demonstrated that all GARCH models were appropriate, with those utilizing the student-t distribution being the most reliable based on three evaluation criteria: RMSE, MAE, and MAPE. Lastly, Hamadu and Ibiwoye (2010) conducted an investigation into the volatility of daily stock returns in Nigerian insurance stocks. They employed daily data from twenty-six insurance companies, spanning from December 15, 2000, to June 9, 2008, as a control dataset and data from June 10, 2008, to September 9, 2008, as an out-of-sample dataset. Their analysis, which considered ARCH (1), GARCH (1, 1), TGARCH (1, 1), and EGARCH (1, 1) models, indicated that EGRACH was the more suitable choice for modeling stock returns. This conclusion was drawn based on superior model-fit assessments using Schwarz and Akaike information criteria, as well as out-of-sample forecasting evaluation using RMSE and Theil's inequality coefficient. Guidolin et al. (2009) studied the effectiveness of both linear and non-linear models in predicting the returns of financial assets in G7 countries were assessed. Their findings revealed that, for asset returns in the United States and the United Kingdom, non-linear models, including threshold autoregressive (TAR) and smooth transition autoregressive (STAR) models, outperformed linear models.

It is cleared that (Aamir et al., 2020; Batool et al., 2022; Fraz et al., 2022b; Zaffar & Hussain, 2022) used linear, nonlinear, regime switching and machine learning models for KSE-100 stock returns but they did not used hybrid models. Similarly, (Midilic; 2020; Mallikarjuna and Rao (2019) Mallikarjuna and Rao (2019); Mademlis and Dritsakis., 2021; Agyarko et al., 2023) used hybrid models but they combined only GARCH or ML models with STAR while ARIMA, ANN, SSA models with SETAR models. They did not used the single SETAR and GARCH individually or in hybrid model form. It shows a clear gap and lack in the literature. This study is aimed to fill this gap and gives the best forecast model that can capture the nonlinearity and conditional variances at the same time for both conventional and Islamic stock market returns of PSX.

METHODOLOGY

In this section, we delve into the modeling approaches for returns and introduce the fundamental characteristics of the models applied in this study. Earlier research on stock market returns predominantly employed classic econometric models, which operated under the assumptions of independence between return rate disturbances and the constancy of variances, as noted by Lin (2018). However, as financial theory and empirical research have advanced, a significant discovery has emerged the clustering nature of stock market volatility. This phenomenon suggests that substantial fluctuations often occur together, while minor fluctuations tend to cluster around similar magnitudes. Furthermore, it has been observed that the variance of return's volatility is not static but undergoes continuous changes. Firstly, the features of financial time series data namely fat tails and high peaks of stock returns are confirmed by the Jerque-Bera test for normality. Then, the stationarity in the stock market returns is checked by using the two unit root tests namely ADF and PP tests. Also, the ARCH-LM test is used to verify the presence of ARCH effect. Furthermore, the BDS test for non-linearity is used to confirm the stock returns series are non-linear. After that, Box-Jenkins ARIMA model are estimated based on different lags of order AR and MA. Maximum four lags for AR and MA are used which are verified by correlograms. The best ARIMA (p,q) model for all the stock market returns are found based on (p=q=1) lags. Similarly, the non-linear SETAR models are estimated with number of thresholds from 1 to 4. The ARIMA-GARCH models are also estimated by using the ARIMA (1,1,1) with standard GARCH (1,1). Lastly, the proposed hybrid GARCH-SETAR models are estimated based on conditional variances from the best estimated GRACH (1,1) model and used this conditional variance series in the threshold SETAR model. All the best fitted models are evaluated on the basis of AIC and BIC information criteria's. After finding the best linear, non-linear and hybrid models, the forecasts are estimated based on the one-step-ahead recursive method of forecasting. The data was distributed into two parts for four different time spans to verify the results. Firstly, for model estimation, data used from January 1st 2012 till June 23, 2023 while from June 24, 2023 till June 30, 2023 is used to compare the 7 days forecast performance. Similarly, for 30, 60 and 90 days, the data from January 1st 2012 till May 31, 2023, May 1^{st} 2023 and April 1^{st} 2023 are used respectively for the model estimation while from 1^{st} June, 2023, May 2^{nd} 2023 and April 2^{nd} 2023 till June 30, 2023 are used to compare forecast performance for 30, 60 and 90 days respectively.

All the forecast evaluation are based on RMSE, MAE, MAPE and SMAPE forecast evaluation criteria's.

ARIMA Models

The combination of AR (p) model and MA (q) model formed of ARMA (m, n) model which expressed as

$\gamma_t = \mu + \varnothing_1 \gamma_{t-1} + \varnothing_2 \gamma_{t-2} + \cdots + \varnothing_m \gamma_{t-m} + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \cdots + \theta_m u_{t-n} + u_t$..................1

Or in sigma notation

$y_t = C + \sum_{i=1}^{m} \varnothing_i y_{t-i} + \sum_{j=1}^{n} \theta_j \varepsilon_{t-j}$..2

Where y_t is the daily stock market prices, C is a constant term, $\varnothing_i$ are the parameter of the autoregressive component of order p, θ_j is the parameters of moving average component of order q, and ε_t is the error term at time t. The order p and q are non-negative integers.

GARCH Models

The ARCH and GARCH models are frequently used to analyze time-varying volatility. The ARCH model, developed by (Engle, 1982) is used to predict the conditional variance of series of returns. The GARCH model is an extension of ARCH model developed by (Bollerslev, 1986). GARCH models assume a specific pattern in the variance of the error term. Compared to the ARCH model, the GARCH model is more concise and requires fewer parameters. There are two parts that consist in GARCH model which are mean equation, y_t ; and variance equation, ${\sigma^2}_t$. The general form for GARCH (p, q) model can be written as follows:

${\sigma^2}_t$=η+$\sum_{i=1}^{p} \beta_i {\sigma^2}_{t-1}$+$\sum_{j=1}^{q} \alpha_j {\varepsilon^2}_{t-j}$..3

where η is the long-run volatility with condition η >0 , $\beta_i \geq$ 0; i = 1, ... , p and $\alpha_j \geq$ 0; j = 1, ... , q . If $\beta_i + \alpha_j$ < 1, then GARCH (p, q) model is covariance stationary. From the general form of GARCH (p, q) model, the GARCH (1, 1) model can defined as

${\sigma^2}_t$=$\eta + \beta {\sigma^2}_{t-1} + \alpha {\varepsilon^2}_{t-1}$ 4

ARIMA-GARCH Models

The ARIMA-GARCH model integrates the linear ARIMA component with the GARCH variance modeling, enabling the modelling of both conditional mean and conditional variance. This approach is versatile, adaptable to various models (Grachev, 2017). Furthermore, the ARIMA-GARCH mixture model bears similarity to the AR-GARCH mixture model introduced by (Wong et al., 1998)

SETAR Models

For a time series i.e. x_t, the self-exciting threshold model known as SETAR can be written as:

$B(L) \bullet x_t + u_t$ if $x_{t-k} \leq x$……………………………………………………5

$(L) \bullet {}_t + v_t$ if $x_{t-k} > x$……………………………………………………6

In the context of a two-regime self-exciting threshold autoregressive process, where "u_t" and "v_t" represent stochastic white noise, "$B(L)$" and "(L)" denote autoregressive polynomials, "k" represents the delay, and "x" signifies the threshold. The choice of delay values is determined by minimizing the sum of squared errors among values ranging from 1 to 12. As for the threshold values, they are determined by observing variations in the analyzed variable.

Proposed Hybrid GARCH-SETAR model

In this study, proposed GARCH-SETAR hybrid model is used to estimate the best and accurate forecast for the selected stock market returns. The methodology of proposed GARCH-SETAR model can be stated as firstly, check either the returns are stationary. After that, identify the presence of ARCH effect. Then find the best standard GARCH model based on (p,q) lags. The best fitted model is also based on AIC and BIC information criteria. Use the estimated conditional variance data series from selected GARCH model in threshold model i.e. SETAR model. Use threshold value from 1 to 4 and find the best fitted hybrid GARCH-SETAR model. Lastly, use the best GARCH-SETAR model to forecast the returns.

Forecast Performance measures

The precision and accuracy of the forecasts reflects the effectiveness of a forecasting model in predicting the stock market returns. Four forecast accuracy measures namely root mean square error (RMSE), mean absolute percentage error (MAPE), symmetric mean absolute percentage error (SMAPE) and mean absolute error (MAE) are employed to assess the appropriateness of the models in this study.

$MAE = \frac{1}{N}\sum_{t=1}^{N}(y_{t+s} - f_{t,s})$……………………………………………………7

$MAPE = 100\, x \frac{1}{N}\sum_{t=1}^{N}\left(\frac{y_{t+s} - f_{t,s}}{y_{t+s}}\right)$……………………………………………8

$RMSE = \sqrt{\sum\left(\frac{Y - \widehat{Y}}{h}\right)}\sum_{t=1}^{N}(y_{t+s} - f_{t,s})$……………………………………………9

$SMAPE = 100\, x\, \frac{1}{N}\sum_{t=1}^{N}\left(\frac{y_{t+s} - f_{t,s}}{y_{t+s} + f_{t,s}/2}\right)$…………………………………………10

Data Collection

Daily data of Islamic and conventional stock market indices namely KMI-30, KSE-30 and KSE-100 of PSX has been taken from yahoo finance.com. The data starts from January 1, 2012, and ends on June 30, 2023. Firstly, the stock market returns are calculated as the compound returns on day 't' and 'p_{t-1}'' represents the index on preceding day 't-1' i.e.:

Returns = R_t = ln (p_t/p_{t-1})...11

RESULTS AND DISCUSSION

Prior to analyzing the stock market returns data, it is essential to gain insight into the fundamental statistical characteristics of the time series data. Figure 1 presents the daily KMI-30, KSE-30 and KSE-100 stock market and returns respectively. All the three stock markets a high fluctuation in the year 2016-17. It shows that all the three stock markets in Pakistan gained huge profit. The reason behind is may be due to the start of CPEC project between China and Pakistan which was officially signed in April 2015. After that, it can be seen the all the stock market of Pakistan started to increase drastically. In the start of 2018, the stock markets of PSX declined drastically but they again started to gain profit in the start of 2019. Unfortunately, in the endo 2019, all the stock markets moves to loss due to Covid-19 pandemic. But after the mid of 2020, the markets started to gain good profits and became stable from 2021 with a very few decline. Also, a suitable time trend can be observed from the stock returns plots. Moreover, the stock returns plots shows indication of clustered patterns and varying variance i.e. high volatility.

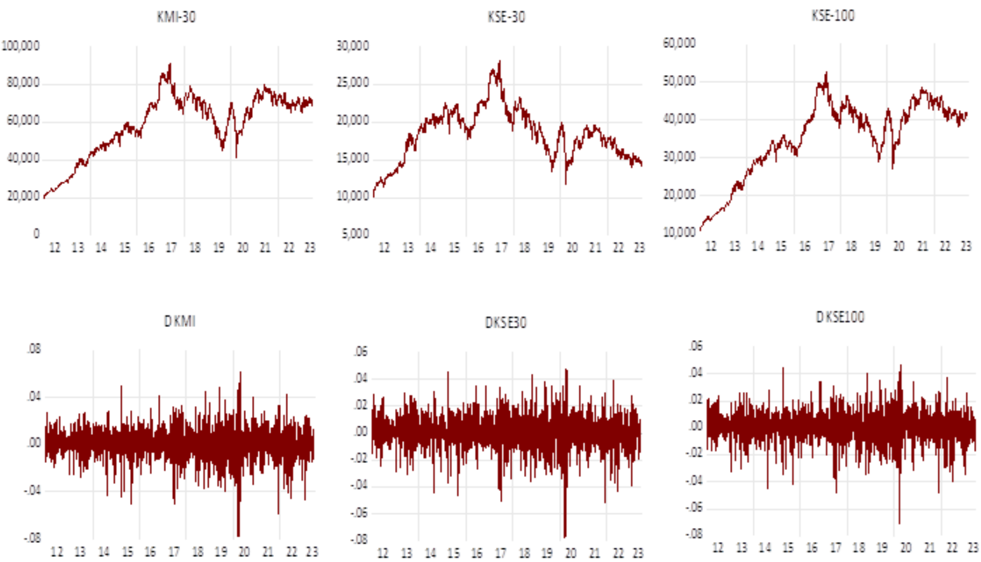

Figure 1: *Stock markets and returns plots*

Data source: Yahoo finance.com

Table 1.
Descriptive Statistics and ARCH Test

Stock Mar-kets	Mean	Std. Dev.	Maximu	Minimun	Skewnes	Kurtosis	Jarque-Bera	ARCH-LM
DKMI30	0.000298	0.00981	0.06193	-0.07831	-0.4567	10.40364	9731.476*	
93.1889*								
93.1889*								
DKSE30	0.000085	0.00935	0.04727	-0.07780	-0.5649	10.44589	9918.56*	116.3708*
DKSE100	0.000309	0.00841	0.0468	-0.07102	-0.6475	10.77506	10864.77*	97.9965*

Author's estimation

According to the findings from Table 1, the average returns series of KMI-30 is 0.0298%, KSE-30 is 0.0085% and KSE-100 is 0.0309% which are very close to zero. It is because the stock returns series typically exhibit a tendency to regress or move back towards the long-term value. The range between the highest and lowest values of KMI-30, KSE-30 and KSE-100 stock returns are 0.1402, 0.1250 and 0.1178 respectively.

Additionally, the standard deviation is 0.98%, 0.93% and 0.84%, respectively, these findings indicates a relatively high level of volatility in the stock market during the selected sample period. The negative skewness values are attributed to the asymmetric tails' negative inclination. Moreover, the kurtosis 10.40, 10.44 and 10.77 for KMI-30, KSE-30 and KSE-100 stock returns respectively , significantly exceed +3 which suggests that the distribution of all the stock returns exhibit fat tails and sharp peak features. Furthermore, none of the returns distribution follows normal distribution pattern, indicating skewed distribution based on Jarque-Bera statistics. It is also necessary to verify the presence of heteroscedasticity in the time series data. The ARCH-LM test also indicates the presence of ARCH effect in all stock markets data. These results ensures that GARCH models can be used in this study.

Table 2.
Unit root tests results

Stock markets	Unit root tests	At Level					
		Intercept	Trend & Intercept	None	Intercept	Trend & Intercept	None
KMI-30	ADF	-2.271	-2.143	0.573	-61.505*	-61.518*	-61.489*
	PP	-2.269	-2.144	0.573	-61.509*	-61.505*	-61.496*
KSE-30	ADF	-2.353	-2.364	-0.078	-60.465*	-60.505*	-60.470*
	PP	-2.458	-2.461	-0.100	-60.795*	-60.775*	-60.801*
KSE-100	ADF	-2.375	-1.964	0.711	-60.712*	-60.738*	-60.688*
	PP	-2.364	-2.069	0.625	-61.092*	-61.093*	-61.090*

Author's estimation

According to unit root test results, all the stock returns i.e. after taking 1^{st} difference, becomes stationary. These findings are based on two unit root tests namely Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests of stationarity (Table 2).

Table 3.
BDS test for non-linearity

Dimension		BDS Statistic	
Stock returns	KMI-30	KSE-30	KSE-100
2	0.01534	0.01499	0.01492
3	0.02364	0.02269	0.02301
4	0.02928	0.02847	0.02967
5	0.03372	0.03224	0.03433
6	0.03377	0.03225	0.03510

Author's estimation

Results presented in Table 3 also shows the presence of non-linearity in all stock market returns. These results are based on Brock-Dechert-Scheinkman (BDS) test. Maximum six dimensions are used to verify the results from BDS test. This evidence revealed that non-linear models i.e. SETAR models can be use in this study.

Table 4.

Forecast performance evaluation for KMI-30 stock market returns

KMI-30 Models	RMSE	MAE	MAPE	SMAPE
7 Days ahead forecast				
ARIMA	918.928	474.380	0.668	0.675
ARIMA-GARCH	908.312	471.128	0.664	0.671
SETAR	910.862	439.723	0.619	0.626
GARCH-SETAR	890.479	412.778	0.581	0.588
30 Days ahead forecast				
ARIMA	615.287	348.165	0.491	0.491
ARIMA-GARCH	625.292	370.248	0.522	0.523
SETAR	615.470	341.556	0.481	0.482
GARCH-SETAR	608.431	337.285	0.475	0.476
60 Days ahead forecast				
ARIMA	545.972	349.495	0.491	0.491
ARIMA-GARCH	549.961	359.507	0.505	0.505
SETAR	546.326	345.383	0.485	0.485
GARCH-SETAR	543.819	342.416	0.481	0.481
90 Days ahead forecast				
ARIMA	515.869	312.422	0.440	0.440
ARIMA-GARCH	521.891	330.431	0.466	0.466
SETAR	516.262	325.081	0.458	0.458
GARCH-SETAR	513.590	303.327	0.427	0.428

Author's estimation

Finally, the empirical evaluation of forecast performance of proposed hybrid GARCH-SETAR model with linear ARIMA, standard ARIMA-GARCH and SETAR models for KMI-30 stock returns are presented in Table 4. According to the findings, the forecast performance of proposed non-linear volatility hybrid GARCH-SETAR model outperforms all other linear, non-linear and volatility models based on four forecast evaluation criteria's namely RMSE, MAE, MAPE and SMAPE respectively. The findings revealed that the proposed hybrid GARCH-SETAR model is the best forecast model for all the selected time span forecast period i.e. 7, 30, 60 and 90 days ahead forecast. It is also revealed that the combination of traditional conditional variance standard GARCH and non-linear threshold SETAR model is the best hybrid model to forecast the Islamic stock market returns.

Table 5.
Forecast performance evaluation for KSE-30 stock market returns

KSE-30 Models	RMSE	MAE	MAPE	SMAPE
7 Days ahead forecast				
ARIMA	188.401	82.797	0.566	0.575
ARIMA-GARCH	188.180	81.848	0.560	0.568
SETAR	191.305	85.153	0.583	0.592
GARCH-SETAR	186.986	75.506	0.516	0.525
30 Days ahead forecast				
ARIMA	120.435	63.127	0.433	0.434
ARIMA-GARCH	120.916	63.924	0.439	0.440
SETAR	122.460	65.509	0.450	0.451
GARCH-SETAR	119.804	61.179	0.420	0.421
60 Days ahead fore-cast				
ARIMA	110.632	65.611	0.446	0.445
ARIMA-GARCH	110.833	66.159	0.449	0.449
SETAR	111.702	69.207	0.470	0.470
GARCH-SETAR	110.814	64.559	0.438	0.438
90 Days ahead forecast				
ARIMA	101.790	57.936	0.391	0.392
ARIMA-GARCH	101.862	58.132	0.393	0.393
SETAR	101.690	57.926	0.386	0.393
GARCH-SETAR	101.667	56.799	0.384	0.384

Author's estimation

Similar to the results from Table 4, the proposed hybrid GARCH-SETAR model outperforms other models for the case of conventional KSE-30 stock market returns based on all forecast evaluation criteria's (Table 5). These findings shows the superior power and ability of combing two different classes of time series modelling techniques i.e. hybrid GARCH-SETAR model.

Table 6.
Forecast performance evaluation for KSE-100 stock market returns

KSE-100 Models	RMSE	MAE	MAPE	SMAPE
7 Days ahead forecast				
ARIMA	491.566	222.136	0.537	0.544
ARIMA-GARCH	496.441	225.344	0.545	0.552
SETAR	499.572	219.928	0.532	0.539
GARCH-SETAR	489.741	196.299	0.474	0.481
30 Days ahead forecast				
ARIMA	311.899	167.174	0.405	0.406
ARIMA-GARCH	316.878	175.731	0.426	0.427
SETAR	313.383	167.299	0.405	0.406
GARCH-SETAR	310.993	160.262	0.388	0.389
60 Days ahead forecast				
ARIMA	271.876	167.811	0.405	0.405
ARIMA-GARCH	273.521	167.930	0.405	0.406
SETAR	272.268	169.660	0.410	0.410
GARCH-SETAR	271.787	163.482	0.395	0.395
90 Days ahead forecast				
ARIMA	249.083	150.591	0.366	0.366
ARIMA-GARCH	250.311	151.534	0.368	0.368
SETAR	245.732	150.078	0.364	0.365
GARCH-SETAR	249.189	145.789	0.354	0.355

Author's estimation

Lastly, the forecast performance of proposed hybrid GARCH-SETAR models are also compared for the case of conventional KSE-100 stock market returns (Table 6). According to the results, the forecasting ability of hybrid GARCH-SETAR model is found to be better as compared to the ARIMA, ARIMA-GARCH, and SETAR models based on all forecast evaluation criteria. These results are similar for all the forecast sample time period i.e. for 7, 30, 60 and 90 days ahead forecasts. According to the literature, over the last few decades, no unique or single model has been identified as suitable for capturing the nonlinear and structural behavior of KSE-100, KSE-30 and KMI-30 stock markets of PSX. The hybrid GARCH-SETAR model can serve as a trademark model to forecast for both Islamic and the conventional stock market returns of Pakistan. The findings of this empirical research study are consistent with those of Siu and Elliot (2022), Naik and Mohan (2021), Midilic (2020), Fathian et al. (2019) and Selmi et al. (2015) who also suggested using the hybrid models. However, the results of this study contradict with the findings of M Mallikarjuna and Rao (2019), who concluded that no unique individual or hybrid model can be the best for every stock market return.

CONCLUSION

The Islamic and conventional stock markets in Pakistan, represented by KMI-30, KSE-30 and KSE-100, exhibit a dynamic nature marked by ongoing fluctuations and uncertainties. These stock markets have consistently demonstrated high volatility since their inception, characterized by frequent and significant fluctuations. Forecasting of such stock market returns is a classical task. In this study, linear and non-linear time series models are used to estimate and forecast the selected daily stock market returns from January 2012 to June 2023. Firstly, the stationarity was checked using the ADF and PP unit root tests. Subsequently, the presence of ARCH effects and non-linearity in the data was verified based on the ARCH-LM and BDS tests respectively. The best estimated linear traditional Box-Jenkins ARIMA, non-linear threshold SETAR and ARIMA-GARCH models were selected based on the AIC and BIC information criteria. Furthermore, the proposed hybrid GARCH-SETAR model was estimated using the conditional variances series derived from the standard GARCH model and used as a benchmark series in the threshold SETAR model. The best hybrid GARCH-SETAR model was also selected based on AIC and BIC criteria. Furthermore, to compare the forecast performance, a one-step-ahead recursive forecast method was utilized. The forecast performance of the proposed hybrid GARCH-SETAR model and other estimated models was compared after dividing the stock returns data into estimation and forecast comparison parts. Forecasts of 7, 30, 60 and 90 days were estimated to verify the results. Empirical findings revealed that the forecast performance of the proposed hybrid GARCH-SETAR model outperformed all other selected models for both Islamic and conventional stock markets, namely KMI-30, KSE-30 and KSE-100 based on RMSE, MAE, MAPE and SMAPE forecast evaluation criteria. These findings can assist both local and international investors, as well as economists in forecasting the stock market returns, whether in Islamic or conventional stock markets of Pakistan.

REFERENCES

Aamir, M., Ali, H., Khalil, U., & Bilal, M. (2020). Comparative analysis of alternative time series models in forecasting Karachi stock exchange. *Ilkogretim Online*, *19*(4), 7981–8002.

Agyarko, K., Frempong, N. K., & Wiah, E. N. (2023). Hybrid model for stock market volatility. *Journal of Probability and Statistics*.

Ardia, D., Bluteau, K., Boudt, K., & Catania, L. (2018). Forecasting risk with Markov-switching GARCH models: A large-scale performance study. *International Journal of Forecasting*, *34*(4), 733–747.

Batool, K., Ahmed, M. F., & Ismail, M. A. (2022). A hybrid model of machine learning model and econometrics' model to predict volatility of KSE-100 Index. *Reviews of Management Sciences*, *4*(1), 225–239.

Bildirici, M., Onat, I. Ş., & Ersin, Ö. Ö. (2023). Forecasting BDI Sea Freight Shipment Cost. *VIX Investor Sentiment and MSCI Global Stock Market Indicator Indices: LSTAR-GARCH and LSTAR-APGARCH Models*, *11*, 1242–1242.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, *31*(3), 307–327.

Devi, N. C. (2018). Evaluating the Forecasting Performance of Symmetric and Asymmetric GARCH Models across Stock Markets: Stock Market Returns and Macroeconomic Variables. *Global Journal of Management and Business Research: B Economics and Commerce*, *18*(2), 21–31.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica: Journal of the econometric society*, 987–1007.

Fathian, F., Fard, A. F., Ouarda, T. B., Dinpashoh, Y., & Nadoushani, S. M. (2019). Modeling streamflow time series using nonlinear SETAR-GARCH models. *Journal of Hydrology*, *573*, 82–97.

Fraz, T. R., Fatima, S., & Uddin, M. (2022a). Evaluating the forecast performance of Spline-GARCH model for stock market volatility: A comparative study of GARCH family models. *Indian Journal of Economics and Business*, *21*(2), 31–42.

Fraz, T. R., Fatima, S., & Uddin, M. (2022b). Modeling and forecasting stock market volatility of CPEC founding countries: using nonlinear time series and machine learning models. *JISR management and social sciences & economics*, *20*(1), 1–20.

Fraz, T. R., Iqbal, J., & Uddin, M. (2020). How well do linear and nonlinear time series models' forecasts compete with international economic organizations. *Business & Economic Review*, *12*(3), 23–70.

Grachev, O. Y. (2017).

Guidolin, M., Hyde, S., Mcmillan, D., & Ono, S. (2009). Non-linear predictability in stock and bond returns: When and where is it exploitable? *International Journal of Forecasting*, *25*(2), 373–399.

Guo, T., Song, S., & Ma, W. (2021). Point and interval forecasting of groundwater depth using nonlinear models. *Water Resources Research*, *57*(12), 2021–030209.

Guptha, K. S. K., & Rao, R. (2018). The causal relationship between financial development and economic growth: an experience with BRICS economies. *Journal of Social and Economic Development*, *20*(2), 308–326.

Hamadu, D., & Ibiwoye, A. (2010). Modelling and forecasting the volatility of the daily returns of Nigerian insurance stocks. *International Business Research*, *3*(2), 106–116.

Hamida, H. B. H., & Scalera, F. (2019). Threshold mean reversion and regime changes of cryptocurrencies using SETAR-MSGARCH models. *International Journal of Academic Research in Accounting*, *9*(3), 221–229.

(2012). Retrieved from http://www.investing.com

Lin, Z. (2018). Modelling and forecasting the stock market volatility of SSE Composite Index using GARCH models. *Future Generation Computer Systems*, *79*, 960–972.

Mademlis, D. K., & Dritsakis, N. (2021). Volatility forecasting using hybrid GARCH neural network models: the case of the Italian stock market. *International Journal of Economics and Financial Issues*, *11*(1), 49–49.

Mallikarjuna, M., & Rao, R. P. (2019). (Vol. 5).

Midiliç, M. (2020). Estimation of STAR-GARCH Models with Iteratively Weighted Least Squares. *Computational Economics*, *55*(1), 87–117.

Mubarik, F., & Javid, A. Y. (2016). Modeling and evaluating forecasting of market index volatility: Evidence from Pakistani stock market. *NUML International Journal of Business & Management*, *11*(2), 81–100.

Mustapa, F. H., & Ismail, M. T. (2019). Modelling and forecasting S&P 500 stock prices using hybrid Arima-Garch Model. *Journal of Physics: Conference Series*, *1366*, 12130–12130.

Naik, N., & Mohan, B. R. (2021). Stock Price Volatility Estimation Using Regime Switching Technique-Empirical Study on the. *Indian Stock Market. Mathematics*, *9*(14), 1595–1595.

Omar, A. B., Huang, S., Salameh, A. A., Khurram, H., & Fareed, M. (2022). Stock market forecasting using the random forest and deep neural network models before and during the COVID-19 period. *Frontiers in Environmental Science*, *10*, 917047–917047.

Opoku, E. E. O., Ibrahim, M., & Sare, Y. A. (2019). Foreign direct investment, sectoral effects and economic growth in Africa. *International Economic Journal*, *33*(3), 473–492.

Rizwan, A., & Khursheed, A. (2018). Forecasting islamic stock market volatility: an empirical evidence from Pakistan economy. *UCP Management Review (UCPMR)*, *2*(1), 17–38.

Selmi, N., Chaabene, S., & Hachicha, N. (2015). Forecasting returns on a stock market using Artificial Neural Networks and GARCH family models: Evidence of stock market S & P 500. *Decision Science Letters*, *4*(2), 203–210.

Siu, T. K., & Elliott, R. J. (2021). Bitcoin option pricing with a SETAR-GARCH model. *The European Journal of Finance*, *27*(6), 564–595.

Wong, W., Xu, L., & Yip, F. (1998).

Zaffar, A., & Hussain, S. A. (2022). (Vol. 81).