

Modelling Stock Indexes Volatility of Emerging Markets

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ABSTRACT

This study aims to investigate the use of ARCH (autoregressive conditional heteroscedasticity) family models for forecasting volatility of four regional emerging stock markets i.e. KSE 100, BSE-SENSEX, DSE 20 and SSE Composite index. The ARCH, GARCH, EGARCH, TGARCH and PARCH models are used and the best model is selected on the basis of the Akaike information criterion (AIC) and Schwartz information criterion (SIC) over the sample period covering from January 1996 to December 2015. Empirical evidence suggested on the basis of AIC, TGARCH outperformed other models in case of BSE SENSEX, DSE 20 and SSE COMPOSITE index. TGARCH model is considered as the best closely followed by PARCH model, whereas PARCH is also considered as the best performing model for BSE SENSEX, KSE 100 and SSE COMPOSITE index. Meanwhile, on the basis of SIC, GARCH is the best performing model for BSE SENSEX and SSE COMPOSITE, whereas PARCH and EGARCH for KSE 100 and DSE 20 respectively. This study will help portfolio managers, investors and policy makers to make their investment strategies in these emerging markets accordingly.

Keywords: Volatility, Forecasting, Emerging Markets

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INTRODUCTION

Stock market plays an important role in any countries economy. Lately stock market forecasting (or prediction) is one of the hottest research topics because of its commercial applications owing to the high stakes and the kinds of lucrative benefits that it offers (Majhi et al., 2007). In addition, it is considered as a challenging job of financial time-series forecast and one of the most important issues in finance. The stock market however, has been investigated by numerous researches, is fundamentally non-linear, nonparametric, and dynamic rather complicated environment (Tan et al., 2005). Miao et al. (2007) and Wang (2002) presented that stock market's movements and fluctuations are affected by several factors like firms' policies, political events, general economic conditions, bank rate, commodity price index, bank exchange rate, investors' expectations, institutional investors' preferences, movements of stock market, psychology and behavior of investors, etc. The understanding and the explanation adequate of the returns of stock markets volatility establishes a fundamental to the study of finance. Investigating data generating process in stock returns, modern research has argued that the factors affecting the asset pricing behavior of investors are explained by non-parametric relationships with expected returns. The predicate of returns of stock markets have showed a significant nonlinearity encouraged from an asymmetric process (Nam et al., 2002; Nam & Kim, 2003). Forecasting and prediction of stock market returns in today's volatile markets have become a challenging task and represent a major task for traditional time-series estima-

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tion. Therefore, predicting economics and finance movements is reasonably difficult. In the literature there are a various methods applied to explain the behavior of time series and to accomplish this challenge.

In determining forecasting procedures time series plays an important in understanding the underlying structure of variables in economics. Many time series occurring specially in the engineering and natural sciences cannot be modeled through linear processes. These time series have specific trends that are captured and modeled by non-linear processes. The modeling structure for non-linear process is rather complicated as compared to linear time series. Some of the important non-linear series include bilinear, exponential auto regressive, threshold autoregressive, autoregressive conditional heteroscedastic (ARCH), generalized autoregressive heteroscedastic (GARCH) and random and stochastic coefficient models.

The main objective of this paper is to use the ARCH family models to estimate and forecast volatility of the stock market returns of Pakistan, India, Bangladesh and China. These four countries are the leading countries in Asia region. The data set for Pakistan is based on the daily closing stock price from the KSE 100 index, the Indian data set is from the BSE SENSEX index the data set for Bangladesh is drawn from the DSE 100 index and the Chinese data is drawn from SSE COMPOSITE index. Moreover, among different appropriate candidate models, the best models will be selected on the basis of the Akaike information criterion (AIC) as proposed by Akaike (1974), the Schwarz information criterion (SIC) given by Schwarz (1978). The other objective of this paper is to compare the results and choose the precise method to forecast volatility for these three stock market return series.

This paper is organized in different sections, in first section introduction and objectives of the study are discussed. In second and third sections detailed review of literature and research methodology are discussed respectively. In fourth section, empirical analysis and in fifth section conclusion and future research directions are given.

LITERATURE REVIEW

Over the last two decades volatility in the financial markets gained massive attention and it can be described as the measurement of the variation in the stock price over the time and it is treated as the measurement of risk. French, Schwert and Stambauch (1987) explain the relationship between volatility and stock returns and found the evidence that risk premium is directly related with volatility. However, to directly get volatility is very hard. Busse (1999) witnessed that timing of volatility is an important factor in the returns of mutual funds that has led to higher risk-adjusted returns. Brandt and Jones (2006) argued that financial stock's volatility is predictable and time-varying but estimating the future volatility level is very complex because it is very difficult to find estimators that truly represent the parameters of volatility. Engle (1982) formulated a model known as ARCH model with the variation of conditional variance.

In ARCH model the restricted variance is dependent upon the previous squared error terms of different lags, even at higher lag, one can hold the maximum number of the restricted variance but a higher order indicates the model is comprised of several parameters which makes the estimation work lengthy, difficult and hard to intercept. Later, in 1986 Bollerslev presented the GARCH model to overcome the higher order ARCH problem. The conditional variance is

dependent upon the previous squared errors and restricted variances of the GARCH model. The extension of ARCH through GARCH is very much similar to the extension of the AR to ARMA model. Engle (2001) shared that GARCH (1, 1) is the most easiest and robust model amongst entire volatility family models. Floros (2008) explained volatility and risk in financial markets using daily observation from Israel (TAS-100) index and Egypt (CMA General Index) using GARCH and its variants. Egyptian CMA index was characterized as the most volatile series. Akgiray (1989), Brooks (1996) and Pagan, Schwert (1989) presented that the GARCH models completely fits on the US stocks appropriately. There are several other GARCH extensions like EGARCH (Exponential GARCH) which was proposed by Nelson (1991). Brandt and Jones (2006) used EGARCH model to estimate and predict volatility of S&P 500.

Engle and Bollerslev (1986) suggested Integrated GARCH (IGARCH) model. Ganger, Ding and Engle (1993) for the first time proposed Power GARCH (PGARCH) model. Lucy and Tully (2006) performed PGARCH models on global gold prices to predict its volatility. Glosten, Runkle and Jaganath (1993) and Zakoian (1994) further proposed Threshold GARCH (TGARCH) model. Chiang (2001) used TGARCH model on seven Asian stock markets to measure the relationship between volatility and returns on stocks. Lastly, Engle and Nag (1993) suggested the Quadratic GARCH (QGARCH) model. Rafique and Kashif-ur-Rehman (2011) applied ARCH, GARCH (1, 1) and EGARCH (1, 1) on KSE 100 index to study the clustering volatility, excess kurtosis and fat tails of the time series of Karachi stock exchange. They noted that GARCH (1, 1) fully captured the volatility persistence. By modeling EGARCH (1, 1) “leverage effect” was successfully overcome in KSE 100 index. Magnus and Fosu (2006) predicted the volatility by taking a single index and using the models like RW, GARCH (1, 1), EGARCH (1, 1) and TGARCH (1, 1) on Ghana stock exchange (GSE). Ng and McAleer (2004) performed volatility models like GARCH (1, 1) and TARCH (1, 1) on Nikkei 225 index and S&P 500 Composite index to estimate and forecast the volatility of daily returns. The results suggested that estimation performance of these models were dependent on the data that was used. GARCH (1, 1) seemed to perform better with Nikkei 225 index whereas; TGARCH (1, 1) was a better measure for S&P 500 index.

Dikko et.al (2015) studied volatility of insurance stocks listed on the Nigerian stock exchange using five asymmetric and seven asymmetric models. Out of the ten stocks that were studied, eight of them showed ARCH effect. Furthermore, ARCH (1) model was considered as the best fit on the basis of MAE, RMSE and MAPE. The novelty of the research is, no such research study has been conducted at preliminary level to just identify the best fit model for these four regional emerging stock markets on the basis of too proven criteria: Akaike and Schwartz information criterion. Data and research methodology is covered in next section of the study.

RESEARCH METHODOLOGY

In this research time series data is used of four Asian emerging stock exchanges i.e. Karachi stock exchange 100 index, Bombay stock exchange SENSEX index, Dhaka stock exchange 20 index and Shanghai stock exchange COMPOSITE index. The study covers a sample period of 20 years starting from January 1, 1996 till December 31, 2015 covering almost 5200 trading days. Each data set includes 4844 observation. The test of normality is conducted for all the transformed series. For this purpose, the Jarque-Bera (JB) test (Jarque and Bera, 1987) of normality is used. The JB test statistic is given below,

$$JB = T \left[\frac{S^2}{6} + \frac{(K-3)^2}{24} \right] \quad (1)$$

Where T is the total number of observations, S is the coefficient of skewness and K is the coefficient of kurtosis. To detect the stationarity of time series data that the time series is stable around its mean, two most widely used unit root tests are used (1) Augmented Dickey Fuller (1987), perhaps the most used test (2) Phillip-Perron (1988) test it makes a non-parametric correction to the t-test statistic. The test is robust with respect to unspecified autocorrelation and Heteroscedasticity in the disturbance process of the test equation. In order to confirm whether the time series is stationary the ADF and PP test statistic should be less than its critical value. Following the autoregressive conditional heteroskedasticity (ARCH) model by Engle (1982), there are many extensions of ARCH model, such as GARCH, EGARCH and TGARCH and PARCH known as ARCH family models.

ARCH (Auto Regressive Conditional Heteroscedasticity)

In 1982 Engle proposed this model, stating that conditional variance of the error/residuals from fitted model at any point in time s dependent on the squared innovations from the past. It is non-linear model which does not take in account that the variances of the error terms are constant and it also elaborates how the variance of the error terms changes. Providing motivation for ARCH family models for time series of financial assets is known as ‘volatility clustering’ or ‘volatility pooling’. Volatility clustering explains the ability of large movements in returns (of any sign) to follow changes and small changes (of any sign) to continue the same trend.

$$h_t = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 \quad (2)$$

$$\varepsilon_t = y_t - x_t' b \quad (3)$$

Whereas q is the order of the moving average ARCH terms and ω and α_i are unknown parameters.

GARCH (Generalized Auto Regressive Conditional Heteroscedasticity)

This model is an extension of ARCH model, proposed by Bollerslev (1986). It suggests that conditional variance of the residual terms also depends on the previous innovations but it is also dependent on the past conditional variances as well. To use GARCH model it is suggested to run it on high frequency data such as daily returns of stock indexes, at lower frequencies the model will not be much effective. If shorter period data is used the result and estimates will not be robust. The primary test before actually forecasting conditional volatility is running Engle’s ARCH test. The equation is given by,

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \quad (4)$$

Where $\alpha_0 > 0$, and $\alpha_i \geq 0$ ($i = 1, 2, \dots, q$), and $\beta_j \geq 0$ ($j = 1, 2, \dots, p$) are the known parameters.

EGARCH (Exponential Generalized Auto Regressive Conditional Heteroscedasticity)

Model EGARCH) model is developed by Nelson (1991). He considered the natural log of conditional variance as linear in some functions of time and past function Z_t to ensure the conditional variance of time series data y_t remains positive. The conditional variance equation can be presented in the following form:

$$\ln(h_t) = \omega + \sum_{i=1}^q \alpha_i \left| \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} \right| + \sum_{j=1}^p \beta_j \ln(h_{t-j}) + \gamma \left| \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} \right| \quad (5)$$

Where, $p \geq 0$; $q > 0$; $\omega > 0$, $\alpha_i > 0$, $i = 1, 2, 3 \dots q$ and $\beta_j \geq 0$, $j = 1, 2, 3 \dots q$.

TGARCH (Threshold Generalized Auto Regressive Conditional Heteroscedasticity)

Zakoian (1994) & Glosten et al. (1993) use the TAR model with an intention of independence than for the asymmetric effect of the “news” (Brooks, 2008). Form of this model is as follows:

$$\ln \sigma_t^2 = \omega + \beta \ln \sigma_{t-1}^2 + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] \quad (6)$$

Where, $\alpha_i \geq 0$ for $i = 1, 2, 3 \dots q$, and $\gamma \geq 0$ for $j = 1, 2, 3 \dots p$.

PARCH (POWER AUTO REGRESSIVE CONDITIONAL HETEROSCEDASTICIY)

The PARCH model is an extension of the GARCH model with an additional term added to account for possible asymmetries (Brooks, 2008). The conditional variance is now given by asymmetries (Brooks, 2008). The conditional variance is now given by,

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1} \quad (7)$$

Where, $\alpha_i \geq 0$ for $i = 1, 2, 3 \dots q$, and $\gamma \geq 0$ for $j = 1, 2, 3 \dots p$.

MODEL SELECTION CRITERION

Residual analyses are carried out for the diagnostic purposes of the models. Among different appropriate candidate models, the best models are selected on the basis of the Akaike information criterion (AIC) as proposed by Akaike (1974) and the Schwarz information criterion (SIC) given by Schwarz (1978).

$$AIC = -2l T + 2k T. \quad (8)$$

$$SIC = -2l T + k \log T / T. \quad (9)$$

Where, k is the number of unknown parameters, T is the size of the series and l is the value of the log likelihood function.

EMPIRICAL ANALYSIS

Since all four of the time series are non-stationary, it first needs to be converted into stationary time series data. In table 1, it can be observed that two out of four stock indexes i.e. DSE 20 and KSE 100 shows leptokurtic behavior and all stock indexes have positive skewness indicating data has long right tail. The p-value of Jarque Bera is less than its critical value of 5% signifying the time series data is non-normal as evident from JB value itself.

Table 1: Descriptive Statistics

Variable	Min	Max	Mean	SD	Kurtosis	Skewness	JB Value	P-value
BSE SENSEX	2600	29681	10784	7499	2.11	0.589	438	0.000
DSE 20	516.4	6092	2081	964.7	4.837	1.261	1965	0.000
KSE 100	765.7	36228	9674	9199.9	3.854	1.27	1449.9	0.000
SSE COMPOSITE	2600	29681	11359	7798	2.029	0.532	418.9	0.000

This table shows the descriptive statistics of all series undertaken in the study.

As per table 2, unit root test was conducted at both the order of integration. Both ADF and PP failed to reject the null hypothesis at zero level of integration indicating that the series have a unit root problem. To address this issue the series are first differenced and at first level of integration both the tests rejected the null hypothesis concluding that the time series is now stationary and possesses a random walk behavior as evident from the table above that p-value is less than 5% of critical value.

Table 2: Unit Root Test

Variables	Order of Integration	ADF Test	PP Test	Hypothesis
BSE SENSEX	I(0)	0.98	0.99	Null hypothesis is not rejected.
	I(1)	0.00	0.00	Null hypothesis is rejected.
DSE 20	I(0)	0.36	0.42	Null hypothesis is not rejected.
	I(1)	0.00	0.00	Null hypothesis is rejected.
KSE 100	I(0)	0.99	0.99	Null hypothesis is not rejected.
	I(1)	0.00	0.00	Null hypothesis is rejected.
SSE COMPOSITE	I(0)	0.94	0.95	Null hypothesis is not rejected.
	I(1)	0.00	0.00	Null hypothesis is rejected.

This table shows random walk behavior among series using Augmented Dickey Fuller and Philip-Perron test.

As per table 2, unit root test was conducted at both the order of integration. Both ADF and PP failed to reject the null hypothesis at zero level of integration indicating that the series have a unit root problem. To address this issue the series are first differenced and at first level of integration both the tests rejected the null hypothesis concluding that the time series is now stationary and possesses a random walk behavior as evident from the table above that p-value is less than 5% of critical value.

ARCH FAMILY MODELS

In this section five different model of normal ARCH family are tested on the four emerging stock indexes time series data i.e. ARCH, GARCH, TARARCH, EGARCH and PARCH.

⁴- Empirical estimations/outputs for the ARCH-type models can be provided through email if required by the readers of the study.

ORDINARY LEAST SQUARES FOR ARCH

First of all ordinary least square was run on first differenced data on all the four emerging stock indexes to determine whether or not there is ARCH effect in the time series. As per the result of output in figure, p-value is less than the critical value of 5% ensuring that ARCH effect is present in the data. Therefore, it is concluded that the current volatility of emerging stock indexes is significantly influenced by their past volatility of their data.

ARCH MODEL

In order to detect Auto Regressive Conditional Heteroscedasticity in all four emerging stock indexes, the p-value of residual $(RESID(-1))^2$ must be less than 5%.

The output of ARCH model indicated that C (constant) is statistically significant for all four emerging stock indexes both in mean and variance equation because its probability value is less than 5%. Furthermore, according to the variance equation the $RESID(-1)^2$ is also statistically significant as the probability is less than 5%, which proves that is ARCH effect. Therefore it can be said that the current volatility is greatly influenced by the past volatility.

Table 1. Autoregressive Conditional Heteroscedasticity

Series	Variable	Coefficient	Std. Error	z-Statistics	Prob.
BSE Sensex Index	Mean Equation				
	@SQRT(GARCH)	0.355431	0.011435	31.08225	0.0000
	C	-42.39465	2.617263	-16.19808	0.0000
	Variance Equation				
	C	13424.66	187.8109	71.47966	0.0000
	RESID(-1) ²	0.864143	0.025105	34.42086	0.0000
DSE 20 Index	Mean Equation				
	@SQRT(GARCH)	0.113635	0.022678	5.010725	0.0000
	C	-2.713963	1.295231	-2.095349	0.0361
	Variance Equation				
	C	60.60081	2.246176	26.97955	0.0000
	RESID(-1) ²	0.153434	0.006901	22.23294	0.0000
	GARCH(-1)	0.855279	0.004834	176.9347	0.0000
KSE 100 Index	Mean Equation				
	@SQRT(GARCH)	0.222970	0.005604	39.78459	0.0000
	C	-11.63397	1.125586	-10.33592	0.0000
	Variance Equation				
	C	5517.100	73.86299	74.69370	0.0000
	RESID(-1) ²	1.296865	0.033492	38.72165	0.0000
SSE Composite Index	Mean Equation				
	@SQRT(GARCH)	0.363912	0.012147	29.96010	0.0000
	C	-44.87930	2.820336	-15.91275	0.0000
	Variance Equation				
	C	14472.27	208.9610	69.25821	0.0000
	RESID(-1) ²	0.854573	0.024250	35.24065	0.0000

GARCH MODEL

To detect Generalized Auto Regressive Conditional Heteroscedasticity in all four emerging stock indexes, the p-value of residual ($\text{RESID}(-1)^2$) must be less than 5%.

The results of GARCH model showed that C (constant) is statistically significant for all emerging stock indexes both in mean and variance equation except for BSE SENSEX index because its probability value is greater than 5%. Furthermore, according to the variance equation the GARCH (-1) and $\text{RESID}(-1)^2$ is also statistically significant as the probability is less than 5%, which proves that is GARCH effect. Therefore it can be said that the current volatility is greatly influenced by the past volatility.

TGARCH MODEL

To detect Generalized Auto Regressive Conditional Heteroscedasticity in all four emerging stock indexes, the p-value of residual must be less than 5%.

The output of TARCH model indicate that none of the C (constant) is statistically significant for all emerging stock indexes both in mean and variance equation except for KSE 100 index because its probability value is less than 5%. Furthermore, according to the variance equation the $\text{RESID}(-1)^2 * (\text{RESID}(-1) < 0)$ and GARCH (-1) is also statistically significant as the probability is less than 5%, which proves that is TARCH effect. Therefore it can be said that the current volatility is greatly influenced by the past volatility.

PARCH MODEL

To detect Power Auto Regressive Conditional Heteroscedasticity in all four emerging stock indexes, the p-value of residual must be less than 5%. The result of PARCH model indicated that none of the C (constant) is statistically significant for all emerging stock indexes both in mean and variance equation except for KSE 100 index because its probability value is less than 5%. As per the variance equation C(3), C(4), C(5) and C(6) are also statistically significant as the probability is less than 5%, which proves that is PARCH effect. Therefore it can be said that the current volatility is greatly influenced by the past volatility.

EGARCH MODEL

In order to detect Exponential Generalized Auto Regressive Conditional Heteroscedasticity in all four emerging stock indexes, the p-value of residuals must be less than 5%. The output of EGARCH model suggested that none of the C (constant) is statistically significant for all emerging stock indexes in mean equation except for KSE 100 index because its probability value is less than 5%. As per the variance equation C(3), C(4), and C(5) are also statistically significant as the probability is less than 5%, which proves that is EGARCH effect. Therefore it can be said that the current volatility is greatly influenced by the past volatility and also there exists an asymmetric behavior in the volatility, meaning that negative shocks affects differently than positive shocks.

BEST FIT MODEL SELECTION

The best fit model for every time series is selected on the basis of Akaike (1974), and Schwarz (1978) information criterion. The model with the lowest information criterion value is considered as the best performing model for that series as given in table 3 below.

Table 3a: Model Selection Criterion – Akaike Information Criterion

Index	ARCH	GARCH	TGARCH	PARCH	EGARCH
BSE SENSEX	12.941	12.320	12.319	12.319	12.323
DSE 20	10.080	9.565	9.565	9.559	9.560
KSE 100	12.247	11.544	11.544	11.4927	11.526
SSE COMPOSITE	13.014	12.404	12.404	12.404	12.406

This table shows selection criteria undertaking Akaike information criterion.

Table 3b: Model Selection Criterion – Schwartz Information Criterion

Index	ARCH	GARCH	TGARCH	PARCH	EGARCH
BSE SENSEX	12.946	12.326	12.327	12.329	12.331
DSE 20	10.080	9.571	9.572	9.568	9.567
KSE 100	12.252	11.550	11.552	11.502	11.534
SSE COMPOSITE	13.019	12.411	12.412	12.413	12.414

This table shows selection criteria undertaking Akaike information criterion.

Empirical evidence suggested on the basis of AIC in table 3a, TGARCH outperformed other models in case of BSE SENSEX, DSE 20 and SSE COMPOSITE index. TGARCH model is considered as the best closely followed by PARCH model, whereas PARCH is also considered as the best performing model for BSE SENSEX, KSE 100 and SSE COMPOSITE index. Meanwhile, on the basis of SIC in table 3b, GARCH is the best performing model for BSE SENSEX and SSE COMPOSITE, whereas PARCH and EGARCH for KSE 100 and DSE 20 respectively.

CONCLUSION & AREA OF FURTHER RESEARCH

In this particular study, ARCH family models are applied on four regional emerging stock indexes, namely BSE SENSEX index of India, DSE 20 index of Bangladesh, KSE 100 index of Pakistan and SSE COMPOSITE index of China. Five different volatility models were used to for study purpose, these are, ARCH, GARCH, TGARCH, PARCH and EGARCH. The best performing models have been selected on the basis Akaike Information Criterion (AIC) and Schwartz Information Criterion (SIC). The model with the lowest value was considered as the best fit model. The daily time series data were used for the indexes. The data includes 4844 observations. The study covers a sample period of 20 years starting from January 1, 1996 till December 31, 2015 covering almost 5200 trading days. Out of the four emerging stock indexes two of them had excess positive kurtosis i.e. DSE 20 index and KSE 100 index, remaining two indexes had platykurtic behavior. All four mentioned indexes showed long positive tails. Time series data was also not normally distributed and had unit root problem which was catered through first differencing. In order to run ARCH family models time series must satisfy three conditions. Volatility clustering should be in the series, data should have fatter

tails and that time series data should be of greater frequency. The outcomes of ARCH model indicate that $\text{RESID}(-1)^2$ is statistically significant for all four emerging stock indexes both in mean and variance equation proving that the current volatility is greatly influenced by the past volatility.

The results of GARCH also showed that the variance equation of GARCH (-1) is also statistically significant signifying that the current volatility risk is greatly influenced by the past square residual and also $\text{RESID}(-1)^2$ is statistically significant for all the stock indexes. Outputs of TARARCH model showed that according to the variance equation the $\text{RESID}(-1)^2 * (\text{RESID}(-1) < 0)$ and GARCH (-1) is also statistically significant therefore it can be said that the current volatility is greatly influenced by the past volatility. Results of PARARCH model indicates that the variance equation C(3), C(4), C(5) and C(6) are all statistically, implying that the current volatility is greatly influenced by the past volatility. In addition the outputs of EGARCH indicated the variance equation C (3), C (4), and C (5) is also statistically significant. Therefore it can be said that the current volatility is greatly influenced by the past volatility and also there exists an asymmetric behavior in the volatility, meaning that negative shocks affects differently than positive shocks.

Empirical evidence suggested on the basis of AIC, TGARCH outperformed other models in case of BSE SENSEX, DSE 20 and SSE COMPOSITE index. TGARCH model is considered as the best closely followed by PARARCH model, whereas PARARCH is also considered as the best performing model for BSE SENSEX, KSE 100 and SSE COMPOSITE index. Meanwhile, on the basis of SIC, GARCH is the best performing model for BSE SENSEX and SSE COMPOSITE, whereas PARARCH and EGARCH for KSE 100 and DSE 20 respectively.

As per the findings it is advised to analyst, researchers and investors to bear this in mind that the bad shocks in the economy or the stock market can significantly impact the volatility in the event of an economic crisis. For future research, different alternative time series models to cater volatility like, multivariate time series models and stochastic volatility models can be considered.

Moreover, in this paper daily closing data was used for a period of 20 year to forecast the volatility. According to ZHOU (1996) tick by tick, minute by minute data will be more effective in measuring and forecasting volatility and applying ARCH models or other relevant volatility models. The results of the study help practitioners, investors, portfolio managers and other stakeholders to consider the best fit models while testing and forecasting the stock returns volatility in the undertaken stock markets.

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